Risk Aversion and the Response of the Macroeconomy to Uncertainty Shocks

Lorenzo Bretscher
LSE*

Alex Hsu
Georgia Institute of Technology†

Andrea Tamoni
LSE‡

First draft: June 2017. This draft: November 17, 2017

Abstract

Degree of risk aversion (RA) determines the impact of second moment shocks in DSGE models featuring stochastic volatility. Ceteris paribus, higher risk aversion leads to stronger responses of macroeconomic variables to volatility shocks, in contrast to the Tallarini (2000) irrelevance result, which still holds with respect to level shocks. The output, consumption, and investment responses roughly double in our model following volatility shocks of the same magnitude as RA increases from 5 to 15, making volatility shocks as economically significant as level shocks in the model. Our result adds another dimension of complication in extending macro-finance models that employ stochastic volatility, such as Bansal and Yaron (2004), from endowment economies to full general equilibrium as macroeconomic and asset pricing moments need to be calibrated simultaneously.

JEL classification: C32, C63, E32, E44
Keywords: Risk Aversion, Uncertainty, Dynamic economies.

*Department of Finance, Email: l.p.bretscher@lse.ac.uk
†Scheller College of Business, Email: alex.hsu@scheller.gatech.edu
‡Department of Finance, Email: a.g.tamoni@lse.ac.uk

We thank Nick Bloom, Sulkhan Chavleishvili (Discussant), and Jesus Fernández-Villaverde for valuable comments. We also thank seminar participants at the SITE Conference – Stanford, at the 25th Finance Forum Conference – Universitat Pompeu Fabra, and at the 49th Money, Macro, and Finance Conference – King's College London.

1 Introduction

Risk matters. A growing strand of literature in economics is focused on documenting the effects of volatility shocks (uncertainty) on macroeconomic dynamics in equilibrium settings. Bloom (2009) provides evidence of time-varying second moment to productivity growth causing significant distortions in output and employment. Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011) estimate an open-economy model to demonstrate the impact of real interest rate volatility shocks on a number of macro variables. These papers find that time-varying risk (or uncertainty) can substantially alter the response of the macroeconomy to exogenous variations. If risk matters, then it is straight forward to conclude that the economic agent's attitude towards risk (or risk aversion) should also matter.[1] We show that, within a class of standard DSGE models, not only risk matters to equilibrium outcomes, but perhaps more importantly, the degree of risk aversion determines the magnitude of these outcomes. One significant implication of our finding is that when calibrating these models to economic data in the presence of stochastic volatility, risk aversion should not be considered a free parameter that can be adjusted to fit financial moments.

We choose three different models from the existing literature to demonstrate this point: the small open economy model of Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011) (FGRU (2011) henceforth), the demand shock model of Basu and Bundick (2017), and the New Keynesian model of Andreasen (2012). These models have the feature that all three exhibit stochastic volatility in aggregate shock processes, and can easily accommodate Epstein and Zin (1989) preferences to allow for the separation of risk aversion from the elasticity of intertemporal substitution. We establish the following two main findings. First, risk aversion amplifies the magnitude of the response of macroeconomic quantities to uncertainty shocks. Second, the source of uncertainty is crucial in generating the dynamic

[1] This might appear to be a trivial point, but Tallarini (2000), in a model with Epstein and Zin (1989) recursive preferences and where the IES is unity, numerically demonstrates the insignificance of the degree of risk aversion in generating macroeconomic fluctuations in an equilibrium model.

response of the model such that risk matters.

The first model under study is the small open-economy proposed by FGRU (2011) where the real interest rate process displays time-varying volatility. We replace the CRRA utility function with Epstein-Zin-Weil recursive utility to separate the effect of risk aversion from that of the elasticity of intertemporal substitution (EIS). The second model of interest is Basu and Bundick (2017). The authors employ a standard New-Keynesian model augmented by Epstein-Zin-Weil utilities with level and volatility shocks to the time discount factor (β) to generate observed comovements in components of aggregate output following uncertainty shocks. The third model we examine is the New-Keynesian model proposed by Andreasen (2012) in which stochastic volatility in technology shock is shown to affect the precautionary saving motive of the representative household which in turn increases term premium on government debt. We conclude that in all three models the endogenous responses of the economy due to volatility shocks are amplified when agents display higher level of risk aversion. However, variance decomposition of endogenous variables in these models show that the interaction of uncertainty shocks and risk aversion are much more pronounced in the first two models relative to the last one. This suggests the source of uncertainty is important in the class of DSGE models we consider, and uncertainty shocks to preferences is more effective in driving the dynamic response of macroeconomic quantities.

In a closely related paper, Gourio (2012)[2] examines the joint implication of risk aversion and time-varying risk on macroeconomic dynamics, but we differ in the source of risk in our models. Rather than focusing on the time-varying probability of disaster risk as in Gourio (2012), we explore the interaction between stochastic volatility and risk aversion. In Proposition 3, Gourio (2012) derives the isomorphic relationship between time-varying disaster probability and shocks to the time discount factor. However, uncertainty in the probability of disaster translates to a level shock to β with constant volatility, whereas the FGRU (2011) and Basu and Bundick (2017) models contain stochastic volatility to preference

[2]Gourio (2013) is a follow on paper in similar spirit but applied to credit spreads.

shocks that directly affect the stochastic discount factor.[3]

High-order perturbation techniques have become the standard method for solving DSGE models.[4] It is also well know that risk premiums are unaffected by first-order terms and completely determined by those second- and higher-order terms. In turn, a widespread macro-finance *separation* paradigm, first proposed by Tallarini (2000), suggests that the moments of macroeconomic quantities are not very sensitive to the addition of second-order and higher-order terms. This result is important since it implies that by varying the risk aversion parameter[5] while holding the other parameters of the model constant, one is able to fit the asset pricing facts without compromising the model's ability to fit the macroeconomic data. We show that this macro-finance separation does not hold in DSGE models featuring stochastic volatility. An immediate implication of our finding is that for the class of macro-finance models using stochastic volatility and recursive preferences together to explain asset prices,[6] the task of replicating the asset pricing mechanism in a production economy can prove to be challenging due to the fact that macroeconomic and asset pricing moments are intertwined in the presence of time-varying risk.

We provide evidence that the effects on allocations of the higher order terms are non-trivial. Figure 1 – Panel A shows how the simulated paths of the investment in the Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011) model are affected by higher-order terms.[7] In particular we evaluate the presence of two innovations, the innovation to the level of real interest rate and to its volatility. Even if the general pattern of behavior is

[3]The shock to the real international risk-free rate in FGRU (2011) is a stochastic shock to the expectation of the stochastic discount factor. The demand shock in Basu and Bundick (2017) effectively produces a stochastic discount factor with time-varying β.

[4]See Aruoba, Fernandez-Villaverde and Rubio-Ramirez (2006) for a discussion about perturbation and alternative solution methods.

[5]Risk aversion only appears in the perturbation solution in higher than first-order terms, see Koijen, van Binsbergen, Rubio-Ramrez and Fernandez-Villaverde (2008).

[6]These are the Long-run Risk models, popularized by Bansal and Yaron (2004), and mostly developed in endowment economies.

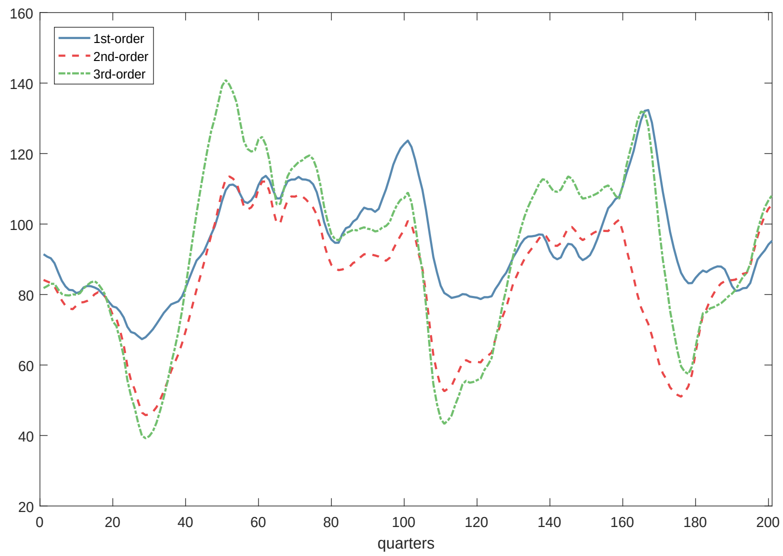
[7]We simulate the model economy of Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011) for 600 periods (after 2000 periods of burn-in to eliminate the effect of initial conditions) at the benchmark calibration parameter values and we follow the results for the deviations of investment with respect to the unconditional mean when we have a first-, second-, and third-order approximation.

similar, there are non-trivial differences. The differences are particularly salient between, on the one hand, the first-order approximation, and on the other hand, the second- and third-order approximations. Importantly the cubic terms in the policy functions are quantitatively significant for real variables. For example, we can see how around periods 160 to 180, in the first-order approximation, investment is rising and then falling; in the second-order approximation, it is falling before recovering; and in the third-order approximation, investment tracks the first-order up to the peak but then it is subject to a more pronounced fall.

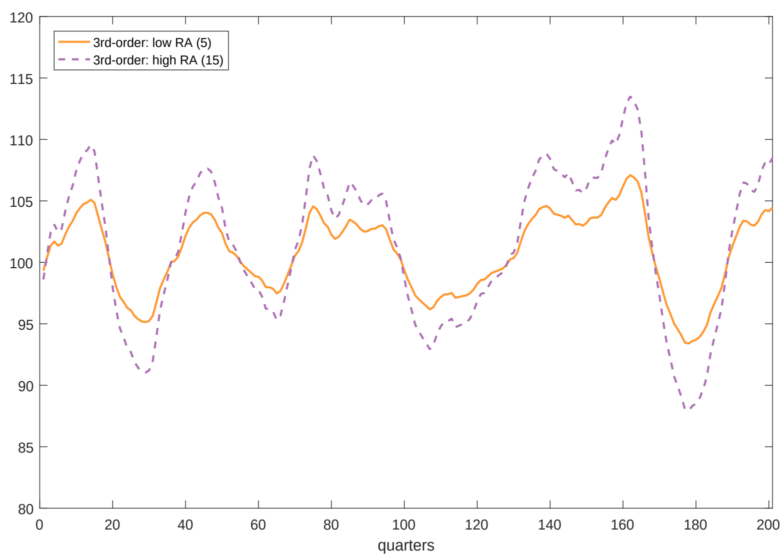
Stochastic volatility is featured in many DSGE models as a way to generate time-variation in risk premia. It is well known that to measure the effects of a volatility increase while keeping the level of the variable unchanged, one needs to obtain a third-order approximation of the policy functions. That is, third-order perturbation is the lowest order perturbation that captures movements in endogenous variables resulting *solely* from changes in uncertainty. Specifically, Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2015) derive analytical coefficient loadings of a second order approximation for generalized policy functions in the presence of stochastic volatility. The solution demonstrates that the second derivatives of the policy functions with respect to stochastic volatility and shocks to stochastic volatility are zeros, implying the policy functions do not load on stochastic volatility under a second order approximation.[8] In this paper we document that within the class of DSGE models featuring stochastic volatility, the response of real macroeconomic quantities to stochastic volatility is tightly linked to the magnitude of the risk aversion. In particular, higher order terms in approximated policy functions have non-trivial effects on the dynamics of macroeconomic quantities, and quantities fall/increase by a larger amount in response to volatility shocks in models with high risk aversion. On the other hand, impulse response functions to level shocks are far less sensitive to the value of risk aversion.

This idea crystallizes in Figure 1 – Panel B. As just discussed, since we are interested in a

[8] See also Andreasen (2012) and Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011).



(a) Evolution of Investment and higher order terms.



(b) Evolution of Investment and Volatility Shocks.

FIGURE 1: This figure plots simulated investment paths. Panel A plots a realization of investment for different order of approximations of the model from FGRU (2011). In each simulation only interest level and interest rate volatility shocks are active. Panel B plots plots a realization of investment for a third-order approximation when only interest rate volatility shocks are active.

volatility increase while keeping the level constant, we have to consider a third-order Taylor expansion of the solution of the model. Panel B represents the contribution of "volatility shocks only" to the investment path in Panel A when we have a third-order approximation of the policy functions. It is apparent that the contribution to investment of changes in the volatility of the real interest rate becomes increasingly important the higher the level of risk aversion. A natural question is whether the investment path in Panel A inherits this dependence of volatility to risk aversion. The answer here is subtle. The path of investment in Panel A can be explained (i) by rate and volatility shocks separately, and (ii) by interaction effect of the rate and volatility shocks. We show in the paper that depending on the model economy and its calibration, the path of investment due to the *simultaneous presence of rate and volatility shocks* (see again Panel A) can be more or less affected by the level of risk aversion. For example, the FGRU (2011) model implies that a higher risk aversion level leads to an increase of overall importance for volatility shocks only at the cost of reducing the contribution of cross-terms of interest rate and volatility shocks (we did not show it in order not to clutter the figure). However, this is not the case for the Born and Pfeifer (2014) recalibration of FGRU (2011), nor for the Basu and Bundick (2017) model.

Our paper is linked with different streams of literature in economics. First, our work is related to the literature on time-varying volatility in finance and macroeconomics. The use of time-varying uncertainty has a long history in finance research. Bansal and Yaron (2004) incorporated long-run risk and recursive preferences in an endowment asset pricing model and showed that stochastic volatility not only generated time-variation in risk premiums but also significantly increased the mean equity risk premium. Recently the role of time-varying uncertainty has been shown to be relevant for macroeconomic variables. Justiniano and Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models with heteroskedastic shocks and show that time-varying volatility helps to explain the Great Moderation between 1984 and 2007. Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012) investigates how uncertainty affects investment and labor demand decisions for the United States; Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011) investigates how interest rate volatility can cause significant fluctuations in output and investment in emerging economies. Our paper complements this line of work by showing that the effects of time-varying volatility on macroeconomic quantities are influenced by the degree of risk aversion; thus asset prices are intertwined with real variables via stochastic volatility and risk-aversion.

2 Illustrative Example

In this section, we write down a simple dynamic model with Epstein and Zin (1989) and Weil (1990) preferences and stochastic volatility to demonstrate the steps in solving such a model using higher order perturbation techniques. The exercise is for illustration purposes only since solving the model in closed form is too burdensome and numerical solutions are well known. The reader familiar with perturbation theory can safely skip this material, and hop to Section 3.

2.1 Economic Environment

We augment a standard real business cycle model with Epstein-Zin-Weil preferences. For simplicity, we abstract away from labor supply. A representative firm takes capital and produces a single consumption good. Capital accumulation is frictionless, meaning there is no adjustment cost other than a constant depreciation each period. We further assume the production technology has decreasing return to scale.

With Epstein-Zin-Weil recursive utilities, the representative agent maximizes lifetime utility by solving the following:

$$
\begin{aligned}
\max \quad & V_t(C_t, V_{t+1}) = \left[(1-\beta)\frac{C_t^{1-\psi}}{1-\psi} + \beta\left\{\mathbb{E}_t\left[V_{t+1}{}^{1-\gamma}\right]\right\}^{\frac{1-\psi}{1-\gamma}}\right]^{\frac{1}{1-\psi}}, \\
\text{s.t.} \quad & C_t + K_{t+1} = e^{z_t}K_t^{\alpha} + (1-\delta)K_t,
\end{aligned}
$$

where ψ is the inverse of elasticity of intertemporal substitution and γ is the coefficient of relative risk aversion. The budget constraint is a combination of the aggregate resource constraint, the production function, and the capital accumulation equation after substituting out investment. The production function is $Y_t = e^{z_t}K_t^{\alpha}$, and z_t is a transitory TFP shock subject to stochastic volatility. Similar to Caldara, Fernández-Villaverde, Rubio-Ramírez and Yao (2012), we specify z_t to follow the exogenous process:

$$
\begin{aligned}
z_t &= \rho z_{t-1} + \chi e^{\sigma_t}\epsilon_t \\
\sigma_t &= (1-\phi)\bar{\sigma} + \phi\sigma_{t-1} + \chi\nu\omega_t,
\end{aligned}
$$

where ϵ_t and ω_t are uncorrelated normal shocks. χ is the perturbation parameter that allows us to turn the stochastic model into a deterministic model, and vice versa.

2.2 Equilibrium Conditions

Optimal decision on consumption and investment result in two standard equilibrium conditions: the stochastic discount factor and the q-investment equation. Respectively,

$$
\begin{aligned}
\frac{\lambda_{t+1}}{\lambda_t} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\left[\frac{V_{t+1}}{\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\psi-\gamma} \quad \text{and} \\
1 &= \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left\{\alpha e^{z_{t+1}}K_{t+1}^{\alpha-1} - (1-\delta)\right\}\right],
\end{aligned}
$$

in which λ_t is the Lagrangian multiplier of the resource constraint and $\alpha e^{z_t} K_{t+1}^{\alpha-1}$ is the marginal productivity of capital. The equilibrium conditions can be combined to get the following rational expectation condition:

$$\mathbb{E}_t \left[\beta C_{t+1}^{-\psi} V_{t+1}^{\psi-\gamma} \left\{ \alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} - (1-\delta) \right\} - C_t^{-\psi} \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{\psi-\gamma}{1-\gamma}} \right] = 0. \tag{1}$$

Denoting $e^{z_t} K_t^{\alpha} + (1-\delta) K_t$ as $f(K_t, z_t)$, the budget constraint implies $C_t = f(K_t, z_t) - K_{t+1}$, which we can plug into Equation (1) to arrive at the following:

$$\begin{aligned} &\mathbb{E}_t \Big[\beta \left\{ f(K_{t+1}, z_{t+1}) - K_{t+2} \right\}^{-\psi} V_{t+1}^{\psi-\gamma} \left\{ \alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} - (1-\delta) \right\} \\ &- \left\{ f(K_t, z_t) - K_{t+1} \right\}^{-\psi} \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{\psi-\gamma}{1-\gamma}} \Big] = 0. \end{aligned} \tag{2}$$

Before we can solve the model using perturbation, we need to include a second equation to completely identify the system of equations. Following Caldara, Fernández-Villaverde, Rubio-Ramírez and Yao (2012), we employ equilibrium condition perturbation (ECP) and include the Epstein Zin value function evaluated at its arguments of the maxima such that

$$V_t - \left[\frac{1-\beta}{1-\psi} \left\{ f(K_t, z_t) - K_{t+1} \right\}^{1-\psi} + \beta \left\{ \mathbb{E}_t \left[{V_{t+1}}^{1-\gamma} \right] \right\}^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}} = 0, \tag{3}$$

after substituting out consumption. Together, Equations (2) and (3) and the shock processes for z_t and σ_t constitute the system of rational expectation equations to be solved by perturbation.

2.3 Solution Strategy

Equations (2) and (3) can be seen as error functions, $\mathbf{F}(s_t)$. They have to hold under any values of K_t and V_t. By the implicit function theorem, the derivatives of the equilibrium conditions also have to hold to be zeros, which allows us to solve for higher order terms in the policy functions of the endogenous variables. As a first step, we rewrite the error functions in logs such that

$$\begin{aligned}
\mathbf{F^1}(s_t) &= \mathbb{E}_t\Big[\beta\left\{f(e^{k_{t+1}}, z_{t+1}) - e^{k_{t+2}}\right\}^{-\psi} e^{(\psi-\gamma)v_{t+1}}\left\{\alpha e^{z_{t+1}+(\alpha-1)k_{t+1}} - (1-\delta)\right\} \\
&\quad - \left\{f(e^{k_t}, z_t) - e^{k_{t+1}}\right\}^{-\psi}\mathbb{E}_t\left[e^{(1-\gamma)v_{t+1}}\right]^{\frac{\psi-\gamma}{1-\gamma}}\Big] = 0 \\
\mathbf{F^2}(s_t) &= e^{v_t} - \left[\frac{1-\beta}{1-\psi}\left\{f(e^{k_t}, z_t) - e^{k_{t+1}}\right\}^{1-\psi} + \beta\left\{\mathbb{E}_t\left[e^{(1-\gamma)v_{t+1}}\right]\right\}^{\frac{1-\psi}{1-\gamma}}\right]^{\frac{1}{1-\psi}} = 0,
\end{aligned}$$

where s_t contains the state variables $(k_t, z_{t-1}, \epsilon_t, \sigma_{t-1}, \omega_t, \chi)$. Policy functions of the endogenous variables can be expressed in terms of s_t following Taylor series expansions around the non-stochastic steady state, $(k_{ss}, 0, 0, \bar{\sigma}, 0, 0)$. Under the second order approximation, policy functions $k_{t+1} = g^1(s_t)$ and $v_t = g^2(s_t)$ can be written as:

$$\begin{aligned}
k_{t+1} &= k_{ss} + g^1_k(k_t - k_{ss}) + g^1_z z_{t-1} + g^1_\epsilon \epsilon_t + g^1_\sigma(\sigma_{t-1} - \bar{\sigma}) + g^1_\omega \omega_t + g^1_\chi \chi \\
&\quad + \frac{1}{2}\left[g^1_{kk}(k_t - k_{ss})^2 + g^1_{zz} z^2_{t-1} + g^1_{\epsilon\epsilon}\epsilon^2_t + g^1_{\sigma\sigma}(\sigma_{t-1} - \bar{\sigma})^2 + g^1_{\omega\omega}\omega^2_t + g^1_{\chi\chi}\chi^2\right] \\
&\quad + g^1_{kz}(k_t - k_{ss})z_{t-1} + g^1_{k\epsilon}(k_t - k_{ss})\epsilon_t + g^1_{k\sigma}(k_t - k_{ss})(\sigma_{t-1} - \bar{\sigma}) + g^1_{k\omega}(k_t - k_{ss})\omega_t \\
&\quad + g^1_{k\chi}(k_t - k_{ss})\chi + g^1_{z\epsilon} z_{t-1}\epsilon_t + g^1_{z\sigma} z_{t-1}(\sigma_{t-1} - \bar{\sigma}) + g^1_{z\omega} z_{t-1}\omega_t + g^1_{z\chi} z_{t-1}\chi \\
&\quad + g^1_{\epsilon\sigma}\epsilon_t(\sigma_{t-1} - \bar{\sigma}) + g^1_{\epsilon\omega}\epsilon_t\omega_t + g^1_{\epsilon\chi}\epsilon_t\chi + g^1_{\sigma\omega}(\sigma_{t-1} - \bar{\sigma})\omega_t + g^1_{\sigma\chi}(\sigma_{t-1} - \bar{\sigma})\chi \\
&\quad + g^1_{\omega\chi}\omega_t\chi, \qquad (4)
\end{aligned}$$

and

$$\begin{aligned}
v_t &= v_{ss} + g^2_k(k_t - k_{ss}) + g^2_z z_{t-1} + g^2_\epsilon \epsilon_t + g^2_\sigma(\sigma_{t-1} - \bar{\sigma}) + g^2_\omega \omega_t + g^2_\chi \chi \\
&\quad + \frac{1}{2}\left[g^2_{kk}(k_t - k_{ss})^2 + g^2_{zz} z^2_{t-1} + g^2_{\epsilon\epsilon}\epsilon^2_t + g^2_{\sigma\sigma}(\sigma_{t-1} - \bar{\sigma})^2 + g^2_{\omega\omega}\omega^2_t + g^2_{\chi\chi}\chi^2\right] \\
&\quad + g^2_{kz}(k_t - k_{ss})z_{t-1} + g^2_{k\epsilon}(k_t - k_{ss})\epsilon_t + g^2_{k\sigma}(k_t - k_{ss})(\sigma_{t-1} - \bar{\sigma}) + g^2_{k\omega}(k_t - k_{ss})\omega_t \\
&\quad + g^2_{k\chi}(k_t - k_{ss})\chi + g^2_{z\epsilon} z_{t-1}\epsilon_t + g^2_{z\sigma} z_{t-1}(\sigma_{t-1} - \bar{\sigma}) + g^2_{z\omega} z_{t-1}\omega_t + g^2_{z\chi} z_{t-1}\chi \\
&\quad + g^2_{\epsilon\sigma}\epsilon_t(\sigma_{t-1} - \bar{\sigma}) + g^2_{\epsilon\omega}\epsilon_t\omega_t + g^2_{\epsilon\chi}\epsilon_t\chi + g^2_{\sigma\omega}(\sigma_{t-1} - \bar{\sigma})\omega_t + g^2_{\sigma\chi}(\sigma_{t-1} - \bar{\sigma})\chi \\
&\quad + g^2_{\omega\chi}\omega_t\chi. \qquad (5)
\end{aligned}$$

Steady state values k_{ss} and v_{ss} can be found by solving the system of equations $\mathbf{F}(k_{ss},0,0,\bar{\sigma},0,0)$. Furthermore, define $z_t = h^1(z_{t-1},\epsilon_t,\sigma_{t-1},\omega_t,\chi) = h^1(s'_t)$ and $\sigma_t = h^2(\sigma_{t-1},\omega_t,\chi) = h^2(s'_t)$ for exposition purposes.

2.4 First Order Approximation

To demonstrate the solution technique, we focus on the error function $\mathbf{F^1}$. Under a first order approximation, only the first order terms in the Taylor series expansion appear (no cross-derivatives). After plugging in $g^1(s_t)$, $g^2(s_t)$, and $h(s'_t)$

$$
\begin{aligned}
&\mathbf{F^1}(s_t)\\
&= \mathbb{E}_t\left[\beta\left\{f(e^{g^1(s_t)},h^1(s'_{t+1})) - e^{g^1(s_{t+1})}\right\}^{-\psi} e^{(\psi-\gamma)g^2(s_{t+1})}\left\{\alpha e^{h^1(s'_{t+1})+(\alpha-1)g^1(s_t)} - (1-\delta)\right\}\right.\\
&\left. - \left\{f(e^{k_t},h^1(s'_t) - e^{g^1(s_t)}\right\}^{-\psi}\mathbb{E}_t\left[e^{(1-\gamma)g^2(s_{t+1})}\right]^{\frac{\psi-\gamma}{1-\gamma}}\right]\\
&= \mathbb{E}_t\left[\beta\left\{f(e^{g^1(s_t)},h^1(h^1(s'_t))) - e^{g^1(g^1(s_t))}\right\}^{-\psi} e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s'_t))}\right.\\
&\left.\left\{\alpha e^{h^1(h^1(s'_t))+(\alpha-1)g^1(s_t)} - (1-\delta)\right\} - \left\{f(e^{k_t},h^1(s'_t) - e^{g^1(s_t)}\right\}^{-\psi}\mathbb{E}_t\left[e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t))}\right]^{\frac{\psi-\gamma}{1-\gamma}}\right]\\
&= 0,
\end{aligned}
$$

and take derivative with respect to k_t:

$$
\begin{aligned}
&\mathbf{F^1}(s_t)|^{k_t}\\
&= \mathbb{E}_t\left[-\beta\psi\left\{f(e^{g^1(s_t)},h^1(h^1(s'_t))) - e^{g^1(g^1(s_t))}\right\}^{(-\psi-1)}\left\{f_k(e^{g^1(s_t)},h^1(h^1(s'_t)))g^1_k - e^{g^1(g^1(s_t))}{g^1_k}^2\right\}\right.\\
&e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s'_t))}\left\{\alpha e^{h^1(h^1(s'_t))+(\alpha-1)g^1(s_t)} - (1-\delta)\right\} + \beta\left\{f(e^{g^1(s_t)},h^1(h^1(s'_t))) - e^{g^1(g^1(s_t))}\right\}^{-\psi}\\
&(\psi-\gamma)e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s'_t))}g^1_k g^2_k\left\{\alpha e^{h^1(h^1(s'_t))+(\alpha-1)g^1(s_t)} - (1-\delta)\right\}\\
&+\beta\left\{f(e^{g^1(s_t)},h^1(h^1(s'_t))) - e^{g^1(g^1(s_t))}\right\}^{-\psi} e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s'_t))}\alpha(\alpha-1)e^{h^1(h^1(s'_t))+(\alpha-1)g^1(s_t)}g^1_k\\
&+\psi\left\{f(e^{k_t},h^1(s'_t)) - e^{g^1(s_t)}\right\}^{-\psi-1}\left\{f_k(e^{k_t},h^1(s'_t)) - e^{g^1(s_t)}g^1_k\right\}\mathbb{E}_t\left[e^{(1-\gamma)g^2(g^1(s_t),h^1(s'_t))}\right]^{\frac{\psi-\gamma}{1-\gamma}}\\
&\left.-\left\{f(e^{k_t},h^1(s'_t)) - e^{g^1(s_t)}\right\}^{-\psi}(\psi-\gamma)\,\mathbb{E}_t\left[e^{(1-\gamma)g^2(g^1(s_t),h^1(s'_t))}\right]^{\frac{\psi-\gamma}{1-\gamma}-1} e^{(1-\gamma)g^2(g^1(s_t),h^1(s'_t))}g^1_k g^2_k\right]\\
&= 0.
\end{aligned}
$$

Evaluating at the steady state, $s_t = (k_{ss}, 0, 0, \bar{\sigma}, 0, 0)$ (i.e. by setting shocks to zeros), the expectations drop out from the above express while $g^1(s_t) = k_{ss}$, $g^2(s_t) = v_{ss}$, and $h(s_t') = 0$:

$$
\begin{aligned}
& \mathbf{F^1}(s_t)|_{ss}^{k_t} \\
= \; & -\beta\psi \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{(-\psi-1)} \left\{ f_k(e^{g^1(s_t)}, h^1(h^1(s_t'))) g_k^1 - e^{g^1(g^1(s_t))} g_k^{1\,2} \right\} \\
& e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s_t'))} \left\{ \alpha e^{h^1(h^1(s_t'))+(\alpha-1)g^1(s_t)} - (1-\delta) \right\} + \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-\psi} \\
& (\psi-\gamma) e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s_t'))} g_k^1 g_k^2 \left\{ \alpha e^{h^1(h^1(s_t'))+(\alpha-1)g^1(s_t)} - (1-\delta) \right\} \\
& + \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-\psi} e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s_t'))} \alpha(\alpha-1) e^{h^1(h^1(s_t'))+(\alpha-1)g^1(s_t)} g_k^1 \\
& + \psi \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-\psi-1} \left\{ f_k(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} g_k^1 \right\} e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s_t'))} \\
& - \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-\psi} (\psi-\gamma)\, e^{(\psi-\gamma)g^2(g^1(s_t),h^1(s_t'))} g_k^1 g_k^2 \\
= \; & -\beta\psi \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{(-\psi-1)} \left\{ f_k(e^{g^1(s_t)}, h^1(h^1(s_t'))) g_k^1 - e^{g^1(g^1(s_t))} g_k^{1\,2} \right\} \\
& \left\{ \alpha e^{h^1(h^1(s_t'))+(\alpha-1)g^1(s_t)} - (1-\delta) \right\} + \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-\psi} \\
& (\psi-\gamma) g_k^1 g_k^2 \left\{ \alpha e^{h^1(h^1(s_t'))+(\alpha-1)g^1(s_t)} - (1-\delta) \right\} \\
& + \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-\psi} \alpha(\alpha-1) e^{h^1(h^1(s_t'))+(\alpha-1)g^1(s_t)} g_k^1 \\
& + \psi \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-\psi-1} \left\{ f_k(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} g_k^1 \right\} \\
& - \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-\psi} (\psi-\gamma)\, g_k^1 g_k^2 \\
= \; & 0,
\end{aligned}
\tag{6}
$$

which is a non-linear equation of two unknowns, g_k^1 and g_k^2.

Next, we move on to the second error function. After replacing $k_{t+1} = g^1(s_t)$, $v_t = g^2(s_t)$, and $z_t = h^1(s_t')$:

$$
\begin{aligned}
& \mathbf{F^2}(s_t) \\
= \; & e^{g^2(s_t)} - \left[\frac{1-\beta}{1-\psi} \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{1-\psi} + \beta \left\{ \mathbb{E}_t \left[e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t'))} \right] \right\}^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}} \\
= \; & 0.
\end{aligned}
$$

Similarly, the derivative of $\mathbf{F^2}(s_t)$ with respect to k_t can be written as

$$
\begin{aligned}
& \mathbf{F^2}(s_t)|^{k_t} \\
= \;& e^{g^2(s_t)} g_k^2 - \left(\frac{1}{1-\psi}\right) \left[\frac{1-\beta}{1-\psi} \left\{f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)}\right\}^{1-\psi}\right. \\
& \left. +\beta \left\{\mathbb{E}_t \left[e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t'))}\right]\right\}^{\frac{1-\psi}{1-\gamma}}\right]^{\frac{\psi}{1-\psi}} \left[(1-\beta)\left\{f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)}\right\}^{-\psi}\right. \\
& \left\{f_k(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} g_k^1\right\} + \beta \left(\frac{1-\psi}{1-\gamma}\right) \left\{\mathbb{E}_t \left[e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t'))}\right]\right\}^{\frac{\gamma-\psi}{1-\gamma}} \\
& \left.(1-\gamma) e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t'))} g_k^1 g_k^2\right] \\
= \;& 0.
\end{aligned}
$$

When evaluated at the steady state, $g^1(s_t) = k_{ss}$, $g^2(s_t) = v_{ss}$, and $h(s_t') = 0$, and the above expression becomes

$$
\begin{aligned}
& \mathbf{F^2}(s_t)|_{ss}^{k_t} \\
= \;& e^{g^2(s_t)} g_k^2 - \left[\frac{1-\beta}{1-\psi} \left\{f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)}\right\}^{1-\psi}\right. \\
& \left. +\beta e^{(1-\psi)g^2(g^1(s_t),h^1(s_t'))}\right]^{\frac{\psi}{1-\psi}} \left[\left(\frac{1-\beta}{1-\psi}\right) \left\{f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)}\right\}^{-\psi}\right. \\
& \left.\left\{f_k(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} g_k^1\right\} + \beta e^{(1-\psi)g^2(g^1(s_t),h^1(s_t'))} g_k^1 g_k^2\right] \\
= \;& 0, \qquad (7)
\end{aligned}
$$

which is again a non-linear equation in g_k^1 and g_k^2. The process is then repeated for each of the remaining state variables $(z_{t-1}, \epsilon_t, \sigma_{t-1}, \omega_t, \chi)$ to generate corresponding error functions evaluated at the steady state, $\mathbf{F}(s_t)|_{ss}^{(z_{t-1},\epsilon_t,\sigma_{t-1},\omega_t,\chi)}$. Combining Equations (6) and (7) with the rest of the errors functions, we can solve for all the unknown coefficient loadings in the policy function, $g^j_{(k,z,\epsilon,\sigma,\omega,\chi)}$ for $j \in \{1,2\}$. It is widely known in the literature that $g_\sigma = g_\omega = 0$ in general, and the coefficient of risk aversion γ does not affect g_k, g_z, g_ϵ, under first order perturbation due to certainty equivalence, consistent with the Tallarini (2000) result.

2.5 Higher Order Approximation

The perturbation solution technique is increasingly complex in several dimensions: the size of the system, the number of state variables, and the order of approximation. Higher order perturbation takes the same approach as that in Section 2.4, but the error functions are 2^{nd}, 3^{rd}, and so on, derivatives of the original system of equations. It is cumbersome to solve for the policy functions analytically even for a two-equation model under first order approximation, as demonstrated previously. For that reason, numerical programs (AIM, Dynare, etc.) are often employed for the purposes of taking derivatives.

In general, the online appendix of Andreasen (2012) shows that the second order terms in the policy function are zeros when they are related to the cross-derivatives of the state variables and the perturbation parameter (χ in the illustrative example here). Numerical simulation of the two-equation model with stochastic volatility used here produces policy functions in line with that observation:

$$\begin{aligned}
k_{t+1} \;=\; & k_{ss} + g^1_k(k_t - k_{ss}) + g^1_z z_{t-1} + g^1_\epsilon \epsilon_t + \cancelto{0}{g^1_\sigma}\,(\sigma_{t-1} - \bar{\sigma}) + \cancelto{0}{g^1_\omega}\,\omega_t + \cancelto{0}{g^1_\chi}\,\chi \\
& + \frac{1}{2}\left[g^1_{kk}(k_t - k_{ss})^2 + g^1_{zz} z^2_{t-1} + \cancelto{0}{g^1_{\epsilon\epsilon}}\,\epsilon^2_t + \cancelto{0}{g^1_{\sigma\sigma}}\,(\sigma_{t-1} - \bar{\sigma})^2 + \cancelto{0}{g^1_{\omega\omega}}\,\omega^2_t + g^1_{\chi\chi}\chi^2 \right] \\
& + g^1_{kz}(k_t - k_{ss})z_{t-1} + g^1_{k\epsilon}(k_t - k_{ss})\epsilon_t + \cancelto{0}{g^1_{k\sigma}}\,(k_t - k_{ss})(\sigma_{t-1} - \bar{\sigma}) + \cancelto{0}{g^1_{k\omega}}\,(k_t - k_{ss})\omega_t \\
& + \cancelto{0}{g^1_{k\chi}}\,(k_t - k_{ss})\chi + g^1_{z\epsilon} z_{t-1}\epsilon_t + \cancelto{0}{g^1_{z\sigma}}\,z_{t-1}(\sigma_{t-1} - \bar{\sigma}) + \cancelto{0}{g^1_{z\omega}}\,z_{t-1}\omega_t + \cancelto{0}{g^1_{z\chi}}\,z_{t-1}\chi \\
& + g^1_{\epsilon\sigma}\epsilon_t(\sigma_{t-1} - \bar{\sigma}) + g^1_{\epsilon\omega}\epsilon_t\omega_t + \cancelto{0}{g^1_{\epsilon\chi}}\,\epsilon_t\chi + \cancelto{0}{g^1_{\sigma\omega}}\,(\sigma_{t-1} - \bar{\sigma})\omega_t + \cancelto{0}{g^1_{\sigma\chi}}\,(\sigma_{t-1} - \bar{\sigma})\chi \\
& + \cancelto{0}{g^1_{\omega\chi}}\,\omega_t\chi, \qquad (8)
\end{aligned}$$

and

$$
\begin{aligned}
v_t = \; & v_{ss} + g^2_k (k_t - k_{ss}) + g^2_z z_{t-1} + g^2_\epsilon \epsilon_t + \cancelto{0}{g^2_\sigma} (\sigma_{t-1} - \bar{\sigma}) + \cancelto{0}{g^2_\omega} \omega_t + \cancelto{0}{g^2_\chi} \chi \\
& + \frac{1}{2} \left[g^2_{kk} (k_t - k_{ss})^2 + g^2_{zz} z^2_{t-1} + \cancelto{0}{g^2_{\epsilon\epsilon}} \epsilon^2_t + \cancelto{0}{g^2_{\sigma\sigma}} (\sigma_{t-1} - \bar{\sigma})^2 + \cancelto{0}{g^2_{\omega\omega}} \omega^2_t + g^2_{\chi\chi} \chi^2 \right] \\
& + g^2_{kz} (k_t - k_{ss}) z_{t-1} + g^2_{k\epsilon} (k_t - k_{ss}) \epsilon_t + \cancelto{0}{g^2_{k\sigma}} (k_t - k_{ss})(\sigma_{t-1} - \bar{\sigma}) + \cancelto{0}{g^2_{k\omega}} (k_t - k_{ss}) \omega_t \\
& + \cancelto{0}{g^2_{k\chi}} (k_t - k_{ss}) \chi + g^2_{z\epsilon} z_{t-1} \epsilon_t + \cancelto{0}{g^2_{z\sigma}} z_{t-1} (\sigma_{t-1} - \bar{\sigma}) + \cancelto{0}{g^2_{z\omega}} z_{t-1} \omega_t + \cancelto{0}{g^2_{z\chi}} z_{t-1} \chi \\
& + g^2_{\epsilon\sigma} \epsilon_t (\sigma_{t-1} - \bar{\sigma}) + g^2_{\epsilon\omega} \epsilon_t \omega_t + \cancelto{0}{g^2_{\epsilon\chi}} \epsilon_t \chi + \cancelto{0}{g^2_{\sigma\omega}} (\sigma_{t-1} - \bar{\sigma}) \omega_t + \cancelto{0}{g^2_{\sigma\chi}} (\sigma_{t-1} - \bar{\sigma}) \chi \\
& + \cancelto{0}{g^2_{\omega\chi}} \omega_t \chi, \qquad (9)
\end{aligned}
$$

where the surviving cross-derivatives (g^j_{kz}, $g^j_{k\epsilon}$, $g^j_{z\epsilon}$, $g^j_{\epsilon\sigma}$, and $g^j_{\epsilon\omega}$ for $j \in \{1, 2\}$) are small relative to the first order terms. This renders stochastic volatility ineffective in driving the dynamics of the model.

Furthermore, van Binsbergen, Fernández-Villaverde, Koijen and Rubio-Ramírez (2012) and Caldara, Fernández-Villaverde, Rubio-Ramírez and Yao (2012) show that in models with Epstein-Zin-Weil preferences, parameter of relative risk aversion (γ) only enters the solution in $g_{\chi\chi}$, generating a constant in the policy rule that can be interpreted as the precautionary behavior toward risk. However, a cursory inspection of the policy functions shows there is no direct interaction between $g_{\chi\chi}$, which contains γ, and stochastic volatility (σ_{t-1}) and the shock to stochastic volatility (ω_t). This implies that a second order perturbation is not enough to explain how increasing risk aversion can amplify the response of the macroeconomic variables to uncertainty shocks. To do so, at least a third order approximation is required because risk aversion enters into all non-zero coefficient loadings in the policy function in front of the square of the perturbation parameter (χ^2), such as $g_{\sigma\chi\chi}$ and $g_{\omega\chi\chi}$. This insight allows us to break the Tallarini (2000) irrelevance result making risk matter in DSGE models.

3 Quantitative effects

This section quantifies the effects of risk aversion on the dynamics of real quantities in DSGE models with time-varying volatility. We consider three models in turn: (1) the Fernandez-Villaverde et al. (2011) model that allows for time-varying volatilities in the real interest rate; (2) the Basu and Bundick (2017) model that features a stochastic volatility shock to the representative household's intertemporal preferences; (3) the Andreasen (2012) model that introduces stochastic volatility in stationary technology shocks. We refer the interested reader to the online Appendix A for details on the model solution, simulation, and computation of IRFs as well as Variance Decomposition.

3.1 The FGRU (2011) Model

The Fernandez-Villaverde et al. (2011) model is suitable for our analysis since it is crafted so that volatility shocks affect real variables like output, consumption, and investment. Next, we briefly describe the Fernandez-Villaverde et al. (2011) model; for the interested reader, a detailed derivation of the model equations, and steady states is available in Fernandez-Villaverde et al. (2011), and hence not repeated here.

The model is a standard small open economy business cycle model calibrated to match data from four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. The small open economy is populated by a representative household. Differently from Fernandez-Villaverde et al. (2011), the preferences of the household are described by an Epstein-Zin-Weil recursive utility function (see Epstein and Zin (1989) and Weil (1990)). We do so because we want to separate the effect of the risk aversion from that of the intertemporal substitution. Trivially, when the risk aversion equals the inverse of the elasticity of substitution we obtain exactly the same results of Fernandez-Villaverde et al. (2011) (see, e.g., Table B.1 in Appendix B.1). The household can invest in two types of assets: the stock of physical capital,

K_t, and an internationally traded bond, D_t. The stock of capital evolves according to the law of motion with adjustment costs:

$$K_{t+1} = (1-\delta) + \left(1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \ .$$

Firms rent capital and labor from households to produce output in a competitive environment according to the technology $Y_t = K_t^{\alpha}\left(e^{X_t} H_t\right)^{1-\alpha}$, where X_t corresponds to a labor-augmenting productivity shock that follows an AR(1) process

$$X_t = \rho_x X_{t-1} + \sigma_x u_{x,t} \ , \tag{10}$$

where $u_{x,t}$ is a normally distributed shock with zero mean and variance equal to one.

Firms maximize profits by equating wages and the rental rate of capital to marginal productivities. Thus,

$$Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1+r_t} + \frac{\Phi_D}{2}\left(D_{t+1} - D_t\right)^2$$

where $\Phi_D > 0$ is a parameter that controls the costs of holding a net foreign asset position.

The model is calibrated to monthly frequency. Following Fernandez-Villaverde et al. (2011), we build quarters of model data, and we report results on a quarterly basis. We refer the interested reader to the online Appendix B.2 for details on the model aggregation.

Finally Fernandez-Villaverde et al. (2011) takes the real interest rate, r_t, as an exogenously given process. We now turn to describe these dynamics.

3.1.1 Stochastic Volatility in Real Interest Rate

The real interest rate, r_t, a country faces on loans denominated in US dollars is decomposed as the international risk-free real rate plus a country–specific spread:

$$r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t}$$

where r is the mean of the international risk-free real rate plus the mean of the country spread; the term $\varepsilon_{tb,t}$, equals the international risk-free real rate subtracted from its mean, and $\varepsilon_{r,t}$ equals the country spread subtracted from its mean. Both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow AR(1) processes described by

$$\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}}u_{tb,t}$$
$$\varepsilon_{r,t} = \rho_r\varepsilon_{r,t-1} + e^{\sigma_{r,t}}u_{r,t} \ ,$$

where both $u_{r,t}$ and $u_{tb,t}$ are normally distributed random variables with mean zero and unit variance. Importantly, the process for interest rates displays stochastic volatility. In particular, the standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ follow an AR(1) process:[9]

$$\sigma_{tb,t} = (1-\rho_{\sigma_{tb}})\,\sigma_{tb} + \rho_{\sigma_{tb}}\sigma_{tb,t-1} + \eta_{tb}u_{\sigma_{tb},t} \tag{11}$$
$$\sigma_{r,t} = (1-\rho_{\sigma_r})\,\sigma_r + \rho_{\sigma_r}\sigma_{r,t-1} + \eta_r u_{\sigma_r,t}, \tag{12}$$

where both $u_{\sigma_{tb},t}$ and $u_{\sigma_r,t}$ are normally distributed random variables with mean zero and unit variance.[10]

Each of the components of the real interest rate is affected by two innovations. For

[9]This specification has been adopted by Justiniano and Primiceri (2008) among others.

[10]Fernandez-Villaverde et al. (2011) re-estimate the process relaxing the assumption that innovations to the country spread and its volatility are uncorrelated, and show that the quantitative patterns remain virtually identical. Thus, in the following analysis we keep the situation without correlation as our benchmark. This also allows us to isolate more clearly the effects of stochastic volatility and risk aversion on real variables.

instance, $\varepsilon_{tb,t}$ is hit by $u_{tb,t}$ and $u_{\sigma_{tb},t}$. The first innovation, $u_{tb,t}$, changes the rate, while the second innovation, $u_{\sigma_{tb},t}$, affects the standard deviation of $u_{tb,t}$. The innovations $u_{r,t}$ and $u_{\sigma_r,t}$ have a similar reading. Section 3.1.3 highlights why it is key to have two separate innovations, one to the level of the interest rate and one to the volatility of the level.

In comparison with the country spread, the international risk-free real rate has both lower average standard deviation of its innovation (σ_{tb} is smaller than σ_r for all four countries) and less stochastic volatility ($\eta_{tb,t}$ is smaller than $\eta_{r,t}$ for all four countries). These relative sizes justify why in our analysis we concentrate only on the innovation to the volatility of the country spread, $u_{\sigma_{tb},t}$, and forget about shocks to the international risk-free real rate. For simplicity, we refer to the innovation $u_{\sigma_{tb},t}$ as the stochastic volatility shock.

3.1.2 Volatility Shocks, Risk Aversion and Macro Dynamics

In this section, we analyze the quantitative implications of the interaction between risk aversion and volatility within the FGRU (2011) model. To judge the importance of volatility shocks for business cycle moments we consider impulse response functions (IRFs) and a variance decomposition.

We first look at the impulse response functions (IRFs) of the model to shocks in the productivity, country spreads, and its volatility. To save space we report the results only for the model calibrated to Argentina. We consider both the original calibration of Fernandez-Villaverde et al. (2011) and the re-calibrated model of Born and Pfeifer (2014).[11] The IRFs from a one standard deviation shock are reported in Figure 2.[12] We plot the IRFs to the three shocks (columns) of output (first row of panels), consumption (second row), investment

[11]We use the same parameters as in Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014); these are reported in Table B.2 for the reader's convenience.

[12]The IRFs must be interpreted as the percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). On the other hand, Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) report deviations from the ergodic mean in absence of shocks, EMAS. See also Appendix B.1 for a comparison between steady state, ergodic mean, and EMAS in the original model of Fernandez-Villaverde et al. (2011), and Appendix A for additional details.

(third row). This figure contains the main point of our paper.

[Insert Figure 2 about here.]

The third column plots the IRFs to a one-standard-deviation shock to the volatility of the Argentinean country spread, $u_{\sigma_{tb},t}$. This column shows that there is a large effect of risk aversion on macro dynamics. Importantly, risk aversion plays an important role only for the the impulse responses from a volatility shock in country spread; indeed the second column shows that IRFs to shocks in the level are hardly affected. Zooming in on the third column, we observe that in response to a volatility shock, output, consumption, and investment fall more in the case of high risk aversion, than in the case of low risk aversion. For example, after a shock to volatility, consumption drops 0.41 percent upon impact for a risk aversion equal to 5; the contraction is larger (0.90 percent at impact) for a risk aversion equal to $= 15$. Similarly, we observe a slow fall in output (after 10 quarters, it falls 0.16 percent) when risk aversion is low. However, for high risk aversion the fall is deeper and more persistent (after 11 quarters years, it falls 0.32 percent). Finally, whereas we observe only a slow decrease of investment (after five quarters it falls 2.18 percent) for low risk aversion, the decrease is substantially larger at 3.98 percent for large risk aversion. Columns $2 - 4$ in Table 1 displays the drops in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion.

[Insert Table 1 about here.]

These IRFs show that the effects of *volatility shocks* on the real economy are intertwined with the magnitude of the risk aversion coefficient. The impact of risk aversion on real quantities is less amplified when one considers instead *level shocks* to productivity or to the real interest rate. These results suggest that increasing the risk aversion parameter to achieve a better fit of model risk premia may have the unintended consequence of affecting the ability of the model to match key macroeconomic moments such as output or investment volatility.[13]

To gauge the contribution of the volatility shocks to aggregate fluctuations for different levels of risk aversion, it is instructive to consider a variance decomposition. Due to the non-linearity of the model and the resulting interaction of shocks, such a variance decomposition cannot be performed analytically. Therefore, we follow Fernandez-Villaverde et al. (2011) and we simulate the model with only a subset of the shocks. In particular, we set the realizations of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. The agents in the model still think that the shocks are distributed by the law of motion that we specified: it just happens that their realizations are zero in the simulation. Table 2 shows the variance decomposition of output, investment, and consumption among different shocks. Each column corresponds to a specific simulation: (1) the benchmark case with all three shocks (productivity, the country spreads and its volatility); (2) when we have a shock only to productivity; (3) when we have a shock to productivity and to the interest rate (with volatility fixed at its unconditional value); (4) when we have a shock only to the interest rate; (5) when we have shocks to interest rate and to volatility; and (6) when we have shocks only to volatility.

[Insert Table 2 about here.]

The last column shows that volatility alone makes a relatively important contribution to the fluctuations of consumption (the standard deviation is 0.75) and investment (standard deviation of 3.08). Increasing the risk aversion almost doubles these contributions. Comparing the last three columns, we can isolate the contribution of interest rate shocks only (column 4), volatility shocks only (column 6), and the interaction effect of the rate and volatility

[13] Two features of the model could potentially affect the results. The FGRU (2011) model assumes that the household faces a cost of holding a net foreign asset position. Importantly, the working paper version Fernandez-Villaverde et al. (2009) quantitatively compared this specification with other ways to close the open economy aspect of the model, and found that the results were if anything, often bigger. Similarly, working capital makes the findings of Fernandez-Villaverde et al. (2011) even stronger. We thus conjecture that our documented interplay between volatility, risk aversion, and real variables is robust to these changes in model specification as well.

shocks (column 5). We first observe that the interaction effect is noticeable: jointly they generate a standard deviation of investment 19.90, while separately they induce standard deviations of 11.63 and 3.08. The difference is accounted for by the cross-terms of interest rate and volatility shocks that appear in the policy function of the agents. Importantly, in the simultaneous presence of both level and volatility shocks to the real rate, increasing the risk aversion does not change the volatility of consumption and investment. However there is a surge in the relative contribution of volatility alone at the expense of cross-terms.[14] We return to this point below.

We next move to consider an alternative calibration of the FGRU (2011) model. In particular, Born and Pfeifer (2014) noted an error in the time aggregation of flow variables, and they show that the model of Fernandez-Villaverde et al. (2011) must be recalibrated. Figure 3 compares the IRFs for the recalibrated corrected model with the IRFs in Fernandez-Villaverde et al. (2011) (these are the same as those shown in Figure 2, and reproduced here for reader's convenience). As noted in Born and Pfeifer (2014), the figure clearly shows that a one standard deviation volatility shock now leads to a larger drop in macro quantities than originally reported in Fernandez-Villaverde et al. (2011). The difference between the two calibrations is further magnified the higher the risk aversion value. Columns $5-7$ in Table 1 displays the drops in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion. We now turn to quantify the effect of the interaction of volatility shocks with risk aversion, as well as the interaction between level and volatility shocks with risk aversion, for business cycle moments in the recalibrated corrected model of Born and Pfeifer (2014).

[Insert Figure 3 about here.]

Table 3 shows the variance decomposition for the alternative calibration proposed by Born

[14] The cross-terms contribution to consumption and investment are $33\% = \frac{7.11-(4.00+0.75)}{7.11}$ and $26\% = \frac{(19.90-(11.63+3.08))}{19.90}$ for low risk aversion. Increasing risk aversion reduces the cross-terms contribution to consumption and investment to $27\% = \frac{6.86-(3.54+1.48)}{6.86}$ and $16\% = \frac{(19.57-(10.54+5.89))}{19.57}$.

and Pfeifer (2014). First, consistent with Born and Pfeifer (2014), we find that in the re-calibrated model that corrects for the time-aggregation, the contribution of volatility shocks to business cycle volatility increase, and more so the higher the risk aversion.[15] Second and more important, by comparing Table 3 with Table 2 an important insight emerges: risk aversion amplifies not only the simulation with volatility shocks only (column 6) but also the simulation where both level and volatility shocks are active (column 5). For example, in the Born and Pfeifer (2014) re-calibrated economy, investment raises by about 28% (18.11/14.19) when risk aversion raises from 5 to 15. On the other hand, the original Fernandez-Villaverde et al. (2011) calibration does not show any sensitivity of investment to risk aversion in a simulation buffeted by level and volatility shocks. This makes us conclude that: (1) volatility shocks are amplified by the magnitude of risk aversion; (2) the amplification effect of risk aversion in a simulation where both level and volatility shocks are active depends on cross-order terms and on the specific calibration of the model.

[Insert Table 3 about here.]

3.1.3 Robustness

Simulation Table 4 checks the stability of our simulations. In particular it shows that our results are robust to an increase in the number of replications, and to removing pruning from the simulations.

[Insert Table 4 about here.]

Stochastic Volatility and Perturbation Solution So far to compute impulse response functions we have relied on the method suggested by Andreasen et al. (2016). This approach

[15]When compared with the benchmark case with all three shocks (column 1), volatility shocks alone (column 6) account for 6 percent of output volatility and 35 percent of investment volatility. By increasing risk aversion, the contribution of volatility shocks to output and investment raises is a remarkable 35% and 67%, respectively.

is based on a pruned state-space system for non-linear DSGE models. The advantage of the pruned state-space system is that it delivers closed-form solutions for the impulse response functions, and it avoids the use of simulation. In particular, Andreasen et al. (2016) provide analytical expressions for the generalized impulse response function (GIRF) when the model is approximated up to third order. In Figure 4 we investigate the effects of pruning and the order of approximation on our results.

[Insert Figure 4 about here.]

The second column in Figure 4 shows that our results are not affected by pruning. When the system is not "pruned", no analytical expression is available and we have to rely on simulation. In this case we follow Andreasen (2012) and define the impulse responses for y_t

$$GIRF(y_{t+l};k) \equiv E_t\left[y_{t+l}; x_t, \{\widetilde{\varepsilon}_{t+i}\}_{i=1}^{l}\right] - E_t\left[y_{t+l}; x_t, \{\varepsilon_{t+i}\}_{i=1}^{l}\right]$$

where $\widetilde{\varepsilon}_{t+1} = \varepsilon_{t+1} + k$ and $\widetilde{\varepsilon}_{t+i} = \varepsilon_{t+i}$ for $i = 0, 1, 2, 3, \ldots, l$. The expectations are evaluated by simulation using 200 draws. Here, ε_{t+i} are drawn from its specified distribution while each value of x_t is taken from a sufficient long sample path of x_t. See also Appendix A for additional details. Impulse responses for a one-standard-deviation shock is given by $k = 1$ while $k = 3$ corresponds to a three-standard-deviation shock.

The third column in Figure 4 investigates how our results are affected by adopting a fifth-order (rather than a third-order) solution for the decision rules. We rely on the approach developed by Fernandez-Villaverde and Levintal (2016) to overcome the computational challenges associated with higher than third-order approximation. The figure shows that fourth- and fifth-order terms are not important for the Fernandez-Villaverde et al. (2011) calibration. Clearly, there might exist parameter values for which these orders are relevant.

Stochastic Volatility: Alternative Functional Forms Figure 5 shows how our results change depending on the functional form used for the stochastic volatility process. This analysis is important because the finance and macro literature have largely specified stochastic volatility processes differently. One can re-write Eq. (11) as follows:

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + m(x_t) u_{r,t} \tag{13}$$

$$x_{t+1} = (1 - \rho_x)\, x + \rho_x x_t + \varepsilon_{x,t+1} \tag{14}$$

where now the innovations are scaled by $m(x_t)$, but we leave unspecified the functional form of $m(\cdot)$. The previous analysis focused on the functional form $m(\cdot) \equiv \exp(\cdot)$ and $x_t = \sigma_{r,t}$. This specification is commonly used in macroeconomics. In contrast, finance papers like to use $m(\cdot) \equiv \sqrt{(\cdot)}$ and $x_t = \sigma^2_{r,t}$.[16] Figure 5 shows that our results are not affected by either functional choice.

[Insert Figure 5 about here.]

Another natural candidate to model time-varying volatility is the ARCH model proposed by Engle (1982) and its various generalizations. In a GARCH model, the conditional volatility is a function of lagged volatility and lagged squared residuals of the level process. Thus, a GARCH process is not driven by separate innovations relative to the level process. On the contrary, the specifications we have analyzed so far admitted two innovations, one to the the country-spread and one to the volatility of the country spread, respectively. In unreported results, we analyze the impulse responses to a real rate shock when stochastic volatility is modeled with GARCH, and we show that the risk aversion does not affect macro dynamics when the time-varying volatility has no separate innovations relative to the level process.

[16] The benefit of the $m(\cdot) \equiv \sqrt{(\cdot)}$ specification is that the stochastic process is still conditionally normal and can be exploited to generate a conditionally log-normal linear approximation that accounts for risk as in Campbell and Shiller (1988). The drawback of this functional form is that it is possible to get a negative standard deviation. The functional form $m(\cdot) \equiv \exp(\cdot)$ ensures the standard deviation remains strictly positive but, as pointed out by Andreasen (2010), has the drawback that the level of the process does not have any moments.

3.2 The Basu-Bundick (2017) Model

Basu and Bundick (2017) build a small-scale dynamic stochastic general equilibrium model with monopolistic competition and sticky prices and show that demand uncertainty can generate a substantial fall in output, consumption, and investment. Basu and Bundick (2017) adopt a recursive structure for intertemporal utility, where a representative household chooses sequences of consumption, C_t, and labor, N_t, to maximize

$$U_t = \left[a_t \left(C_t^{\eta}(1-N_t)^{1-\eta}\right)^{\frac{1-\sigma}{\theta}} + \beta \left(E_t U_{t+1}^{1-\sigma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\sigma}}$$

where $\theta \equiv (1-\sigma)/(1-1/\psi)$, σ determines the coefficient of relative risk aversion, ψ is the intertemporal elasticity of substitution, β is the subjective discount factor, and η determines the Frisch elasticity of labor supply. The model is calibrated to quarterly frequency by matching impulse responses to a one standard deviation increase in the VXO from a vector autoregression. We use the values listed in Basu and Bundick (2017) – Table I; in particular their calibration entails a value $\sigma = 80$.

Next we describe how Basu and Bundick (2017) modeled the demand uncertainty channel.

3.2.1 Stochastic Volatility in household's intertemporal preferences

The coefficient on current utility, a_t, is a preference shock that follows

$$a_t = (1-\rho_a)\, a + \rho_a a_{t-1} + e^{\sigma_{a,t}} \varepsilon_{a,t} \tag{15}$$

$$\sigma_{a,t} = (1-\rho_\sigma)\, \sigma_a + \rho_\sigma \sigma_{a,t-1} + \sigma_\sigma \varepsilon_{\sigma_a,t} \tag{16}$$

where the standard deviation of the preference shock, $\sigma_{a,t}$, follows an independent Normal process (meaning $\varepsilon_{a,t}$ and $\varepsilon_{\sigma_a,t}$ are uncorrelated) to introduce time-varying demand uncertainty into the model.

3.2.2 Volatility Shocks, Risk Aversion and Macro Dynamics

Figures 6(a) and 6(b) show the IRFs to a level and volatility shock to the representative household's intertemporal preferences, respectively.

[Insert Figure 6 about here.]

Figure 6(b) shows that in response to a volatility shock the decline in output, consumption, and investment is stronger the greater the risk aversion. On the other hand, Figure 6(a) shows that IRFs to level shocks are hardly affected by the level of risk aversion. Thus, these IRFs confirm that the effects of *volatility shocks* on the real economy are intertwined with the magnitude of the risk aversion coefficient.

To gauge the contribution of the volatility shocks to aggregate fluctuations for different levels of risk aversion, it is instructive to consider a variance decomposition. Table 2 shows the variance decomposition of output, consumption, investment, and hours among different shocks. Each column corresponds to a specific simulation: (1) the benchmark case with all three shocks (productivity, level and volatility shocks to the representative household's intertemporal preferences); (2) when we have a shock only to productivity; (3) when we have shocks to the level and the volatility of the household's intertemporal preferences; and (4) when we have shocks only to volatility (pure demand uncertainty channel).

[Insert Table 5 about here.]

The last column shows that volatility alone makes a relatively important contribution to the fluctuations of real variables; more importantly increasing the risk aversion almost doubles these contributions. Looking at the second to last column we observe that risk aversion amplifies not only the simulation with volatility shocks (column 5) but also the simulation where both level and volatility shocks are simultaneously active (column 4). For example, doubling risk aversion raises investment by about 15% (5.11/4.43) in a simulation with level and volatility shocks. In short, these results confirm those for the FGRU (2011) economy recalibrated as suggested by Born and Pfeifer (2014): (1) volatility shocks are amplified by the magnitude of risk aversion; (2) the amplification effect of risk aversion is also present, although attenuated, in a simulation where both level and volatility shocks are active.

3.3 The Andreasen (2012) Model

This section investigates the interplay of risk aversion and uncertainty in a model that introduces stochastic volatility in stationary technology shocks. To this end we explore the effects of risk aversion and volatility shocks in the DSGE model proposed by Andreasen (2012), which in turn extends the Rudebusch and Swanson (2012) model. This model is a standard New Keynesian model extended with recursive preferences (see Epstein and Zin (1989) and Weil (1990)). Final output is produced by a perfectly competitive representative firm which uses a continuum of intermediate goods. All intermediate firms produce a slightly differentiated good using

$$Y_t(i) = Z_t A_t \bar{K}^{\theta} N_t(i)^{1-\theta}$$

where $\bar{K}$ and $N_t(i)$ denote physical capital and labor services of the i'th firm, respectively. The variable A_t represents exogenous stationary technology shocks specified below. The variable Z_t captures a deterministic trend in technology, meaning that $\mu_{z,t} \equiv \frac{Z_t}{Z_{t-1}}$ and $\mu_{z,t} = \mu_{z,ss}$ for all t. The model features quadratic price adjustment costs a-la Rotemberg (1982) and the behavior of the central bank is given by a standard Taylor rule. The model is calibrated at quarterly frequency (see Table 1 in Andreasen (2012)). In particular the calibration implies a relative risk-aversion of about 168.

We now turn to the dynamics of technology shock.

3.3.1 Stochastic Volatility in Productivity

The technological process $a_t = \log(A_t)$ evolves according to a stochastic volatility process

$$a_{t+1} = (1-\rho_a)\, a_{ss} + \rho_a a_t + e^{\sigma_{a,t+1}} \varepsilon_{a,t+1} \tag{17}$$

$$\sigma_{a,t+1} = (1-\rho_\sigma)\, \sigma_{a,ss} + \rho_\sigma \sigma_{a,t} + \varepsilon_{\sigma,t+1} \tag{18}$$

with $\varepsilon_{a,t} \sim$ i.i.d.$N(0,1)$, $a_{ss} = \log A_{ss}$ where A_{ss} is the steady state level of A_t, and $\varepsilon_{\sigma,t} \sim$ i.i.d.$N(0,\sigma_\sigma)$. The innovations $\varepsilon_{a,t+1}$ and $\varepsilon_{\sigma,t+1}$ are assumed to be mutually independent at all leads and lags. In words, two independent innovations affect the the level of productivity. The first innovation, $\varepsilon_{a,t+1}$, changes the level of productivity itself, while the second innovation, $\varepsilon_{\sigma,t+1}$, determines the spread of values for the productivity level.

3.3.2 Volatility Shocks, Risk Aversion and Macro Dynamics

Impulse responses from a one standard deviation shock are reported in Figure 7.

[Insert Figure 7 about here.]

The second column in Figure 7 shows that there is a significant effect of risk aversion on macro dynamics. The amplification effect of risk aversion is present only for impulse responses to a volatility shock in technology; responses to level shocks do not display any sensitivity to the risk aversion parameter. Zooming in on the second column, we observe that a higher level of risk aversion generates a more pronounced decline in output, consumption, and hours in response to a volatility shock. Similarly, inflation increases more in the case of high risk aversion. Thus, increasing the risk aversion to achieve a better fit of risk premia may affect the ability of the model to match key macroeconomic moments such as consumption volatility. The Appendix C shows that our conclusion is robust to the pruning scheme, order of approximation and stochastic volatility specification.

It is interesting to compare the variance decomposition obtained from the Andreasen (2012) model (see Table 6) with the variance decompositions of the FGRU (2011) open economy model and the Basu and Bundick (2017) demand shock model (see Tables 2 and 5). The latter two economies are characterized by a much stronger effect of volatility shocks in terms of driving economic dynamics compared with the standard New-Keynesian model with uncertainty in productivity. For example, when rising risk aversion in the Andreasen (2012) economy, the variability of output and consumption due to volatility shocks in productivity increases by 40% and 55%, respectively. Although these increases are still substantial, they are far below those observed for the FGRU (2011) and the Basu and Bundick (2017) economies.[17]

[Insert Table 6 about here.]

Similarly, in the Andreasen (2012) economy, the simulation with both level and volatility shocks is almost unaffected by the level of risk aversion. On the other hand, economies featuring shocks that directly affect the stochastic discount factor display a substantial sensitivity of a simulation with both level and volatility shocks to risk aversion: e.g. Tables 3 and 5 show that by increasing risk aversion, investment raises by 28% (18.11/14.19) and 15% (5.11/4.43), respectively.

To conclude, a careful inspection of the variance decomposition of endogenous variables in the three model considered in the paper show that the interaction of uncertainty shocks and risk aversion is much more pronounced in the FGRU (2011) (and, in particular, its recalibration by Born and Pfeifer (2014)) and the Basu and Bundick (2017) models relative to the Andreasen (2012) one. This suggests the source of uncertainty is important in the class of DSGE models we consider, and uncertainty shocks to preferences is more effective in driving the dynamic response of macroeconomic quantities.

[17] In FGRU (2011) the variability of consumption raises by $97\% = 1.48/0.75 - 1$, whereas in Basu-Bundick (2017) it raises by $105\% = 0.43/0.21 - 1$.

4 Conclusion

Our study shows that, within the class of DSGE models widely used for policy analysis, the response of macroeconomic quantities to volatility shocks is stronger for higher level of risk aversion. IRFs to level shocks are less sensitive to the value of risk aversion. The effect we document are quantitatively important: after a shock to volatility, the higher the risk aversion, the larger and more prolonged the decline in economic activity. On the other hand, these models are much more robust to varying degrees of risk aversion with respect to level shocks in the economy.

By examining the interactions of risk aversion and volatility shocks in three well known DSGE models in the literature, we also document that the impact of volatility shocks is significantly more pronounced in models where uncertainty is directly related to preferences rather than production. Variance decompositions of the FGRU (2011) open economy model and the Basu and Bundick (2017) demand shock model underscore a much stronger effect of volatility shocks in terms of driving economic dynamics compared to the baseline model in Andreasen (2012), which is a standard New-Keynesian model with uncertainty in productivity.

Our results could be relevant for policymakers to consider stochastic volatility, and its interplay with financial quantities via risk-aversion, when implementing fiscal and monetary policy. For future work, we are building a DSGE model with uncertainty on the production side and a representative agent with external habit to investigate the amplification effect of time-varying risk aversion on uncertainty shocks. Potentially, as uncertainty exacerbates the precautionary savings motive of the representative agent, consumption declines more causing endogenous risk aversion to rise. This leads to further decline in consumption next period and thus prolonging the downturn stemming from economic uncertainty.

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Tables

Table 1: IRF Analysis - The Effect of a Volatility Shocks

This table reports displays the drops in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion. Results are shown for both the model by Fernandez-Villaverde et al. (2011) and the recalibrated corrected model by Born and Pfeifer (2014). "Recovery time" is defined as time (closest quarter) it takes for a variable to revert back to its unconditional mean. An example: With a risk aversion of 15, it takes 4 quarters for consumption to revert back to its mean level after a one standard deviation shock in volatility. See also Figure 3.

	FGRU (2011)			BP (2014)		
(1) Risk Aversion	(2) Largest Drop	(3) Time	(4) Recovery Time	(5) Largest Drop	(6) Time	(7) Recovery Time
Panel A: Consumption						
5	-0.410	1	4	-1.125	1	8
15	-0.896	1	4	-2.757	1	7
20	-1.381	1	5	-4.390	1	7
25	-1.866	1	5	-6.022	1	7
Panel B: Investment						
5	-2.183	5	14	-4.230	5	15
15	-3.979	5	15	-9.778	5	14
20	-5.774	5	15	-15.353	4	14
25	-7.570	5	16	-20.957	4	14
Panel C: Output						
5	-0.165	10	22	-0.287	10	26
15	-0.321	11	25	-0.753	11	26
20	-0.481	11	26	-1.2209	11	26
25	-0.642	12	27	-1.6886	11	26

Table 2: Variance Decomposition FGRU (2011) - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks in the model of FGRU (2011) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: only level shocks to the spread and the T-bill rate; fifth column: without TFP shocks; sixth column: only shocks to the volatility of spreads and the T-bill rate.

	(1) All Shocks	(2) TFP only	(3) w/o volatility	(4) Rate level	(5) w/o TFP	(6) Volatility only
Panel A: $\gamma = 5$, Pruning, 200 Replications						
σ_Y	5.25	5.02	5.09	0.64	1.10	0.16
σ_C	7.60	2.63	4.70	4.00	7.11	0.75
σ_I	20.63	5.00	12.71	11.63	19.90	3.08
Panel B: $\gamma = 15$, Pruning, 200 Replications						
σ_Y	5.23	5.01	5.07	0.57	1.07	0.31
σ_C	7.42	2.73	4.38	3.53	6.86	1.48
σ_I	20.42	5.42	11.86	10.54	19.57	5.89

Table 3: Variance Decomposition BP (2014) - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks in the recalibrated model of BP (2014) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: only level shocks to the spread and the T-bill rate; fifth column: without TFP shocks; sixth column: only shocks to the volatility of spreads and the T-bill rate.

	(1) All Shocks	(2) TFP only	(3) w/o volatility	(4) Rate level	(5) w/o TFP	(6) Volatility only
Panel A: $\gamma = 5$, Pruning, 200 Replications						
σ_Y	4.59	4.46	4.49	0.40	0.75	0.27
σ_C	4.39	2.10	2.72	1.76	3.81	1.40
σ_I	15.51	5.84	9.78	7.80	14.19	5.46
Panel B: $\gamma = 15$, Pruning, 200 Replications						
σ_Y	4.61	4.44	4.47	0.38	0.93	0.66
σ_C	5.24	2.40	2.65	1.20	4.63	3.38
σ_I	19.54	6.68	9.85	7.11	18.11	13.10

Table 4: Variance Decomposition FGRU (2011) - Robustness Tests

This table reports the variance decomposition for the different structural shocks in the model of FGRU (2011) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: only level shocks to the spread and the T-bill rate; fifth column: without TFP shocks; sixth column: only shocks to the volatility of spreads and the T-bill rate.

	(1) All Shocks	(2) TFP only	(3) w/o volatility	(4) Rate level	(5) w/o TFP	(6) Volatility only
	Panel A: $\gamma = 15$, Pruning, 1000 Replications					
σ_Y	5.29	5.14	5.18	0.58	1.07	0.31
σ_C	7.60	2.81	4.57	3.63	7.00	1.52
σ_I	20.81	5.55	12.13	10.89	20.01	6.01
	Panel B: $\gamma = 15$, No Pruning, 200 Replications					
σ_Y	5.22	5.01	5.07	0.58	1.01	0.29
σ_C	7.18	2.75	4.46	3.60	6.57	1.46
σ_I	19.01	5.43	11.97	10.63	18.11	5.74

Table 5: Variance Decomposition Basu and Bundick (2017) - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks in the model of Basu and Bundick (2017). First column displays moments obtained from data. Second column: 200 simulations of the model; third column: TFP shocks only; fourth column: without volatility shocks to household discount factor; fifth column: only shocks to the volatility of household discount factor.

	(1) Data	(2) All Shocks	(3) TFP only	(4) w/o TFP	(5) Volatility only
	Panel A: $\gamma = 80$, Pruning, 200 Replications				
σ_Y	1.1	1.06	0.65	0.81	0.40
σ_C	0.7	0.77	0.37	0.66	0.21
σ_I	3.8	4.73	1.52	4.43	0.97
σ_H	1.4	0.80	0.16	0.78	0.25
	Panel B: $\gamma = 160$, Pruning, 200 Replications				
σ_Y	1.1	1.35	0.74	1.12	0.82
σ_C	0.7	0.90	0.41	0.78	0.43
σ_I	3.8	5.44	1.80	5.11	2.03
σ_H	1.4	0.96	0.19	0.94	0.51

Table 6: Variance Decomposition in Andreasen (2012) - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks in the model of Andreasen (2012). First column: 200 simulations of the model; second column: TFP shocks only; third column: without TFP shocks; fourth column: only shocks to the volatility of TFP.

	(1) All Shocks	(2) TFP only	(3) w/o TFP	(4) Volatility only
Panel A: $\gamma = 107.5$, Pruning, 200 Replications				
σ_Y	1.82	1.59	0.92	0.05
σ_C	3.89	3.40	1.96	0.09
σ_H	2.62	2.15	1.44	0.07
Panel B: $\gamma = 167.5$, Pruning, 200 Replications				
σ_Y	1.83	1.60	0.92	0.07
σ_C	3.90	3.41	1.96	0.14
σ_H	2.61	2.15	1.44	0.10

Figures

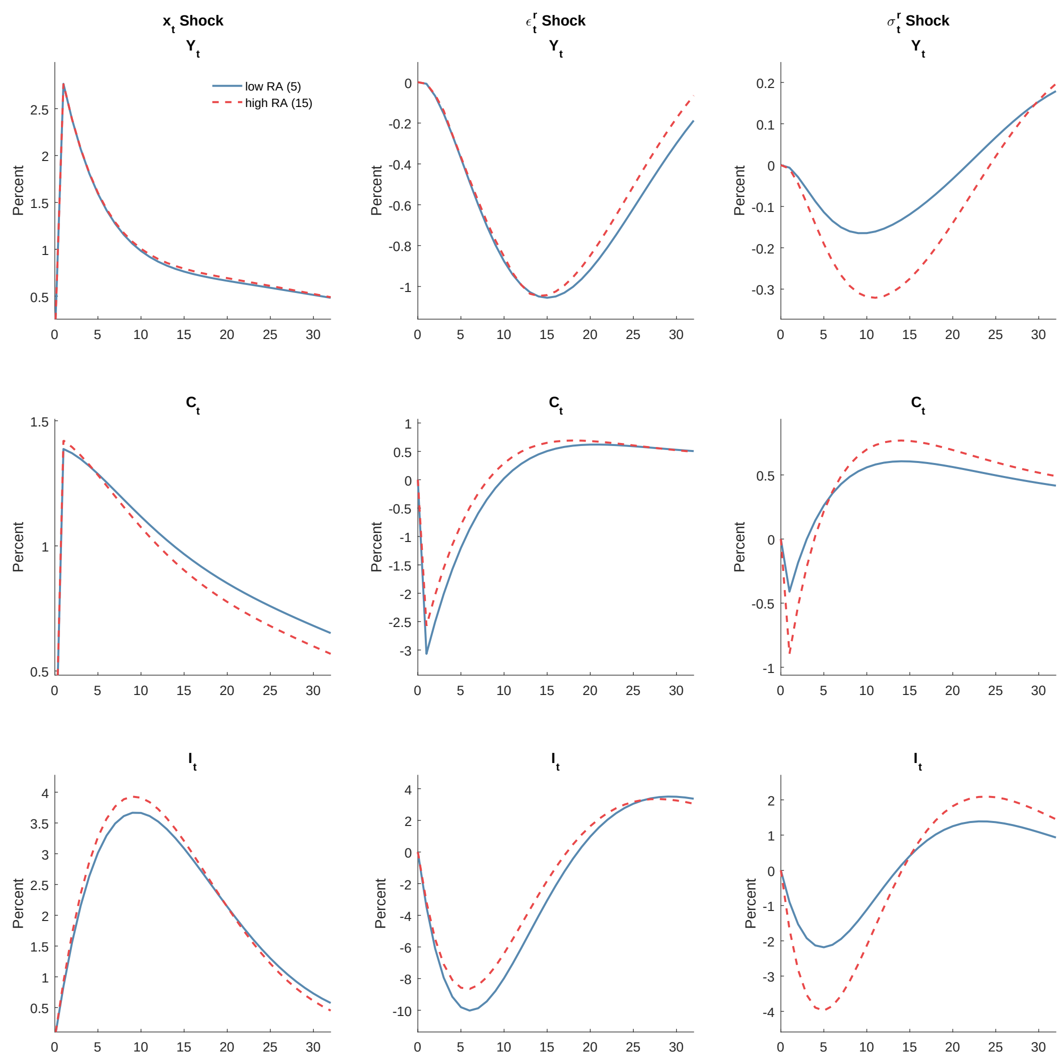


FIGURE 2: **(Unconditional) Impulse Response Functions - FGRU (2011)**

This figure plots the impulse responses for a one standard deviation shock to the (i) technology level (ii) interest rate (iii) conditional volatility in interest rate. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).

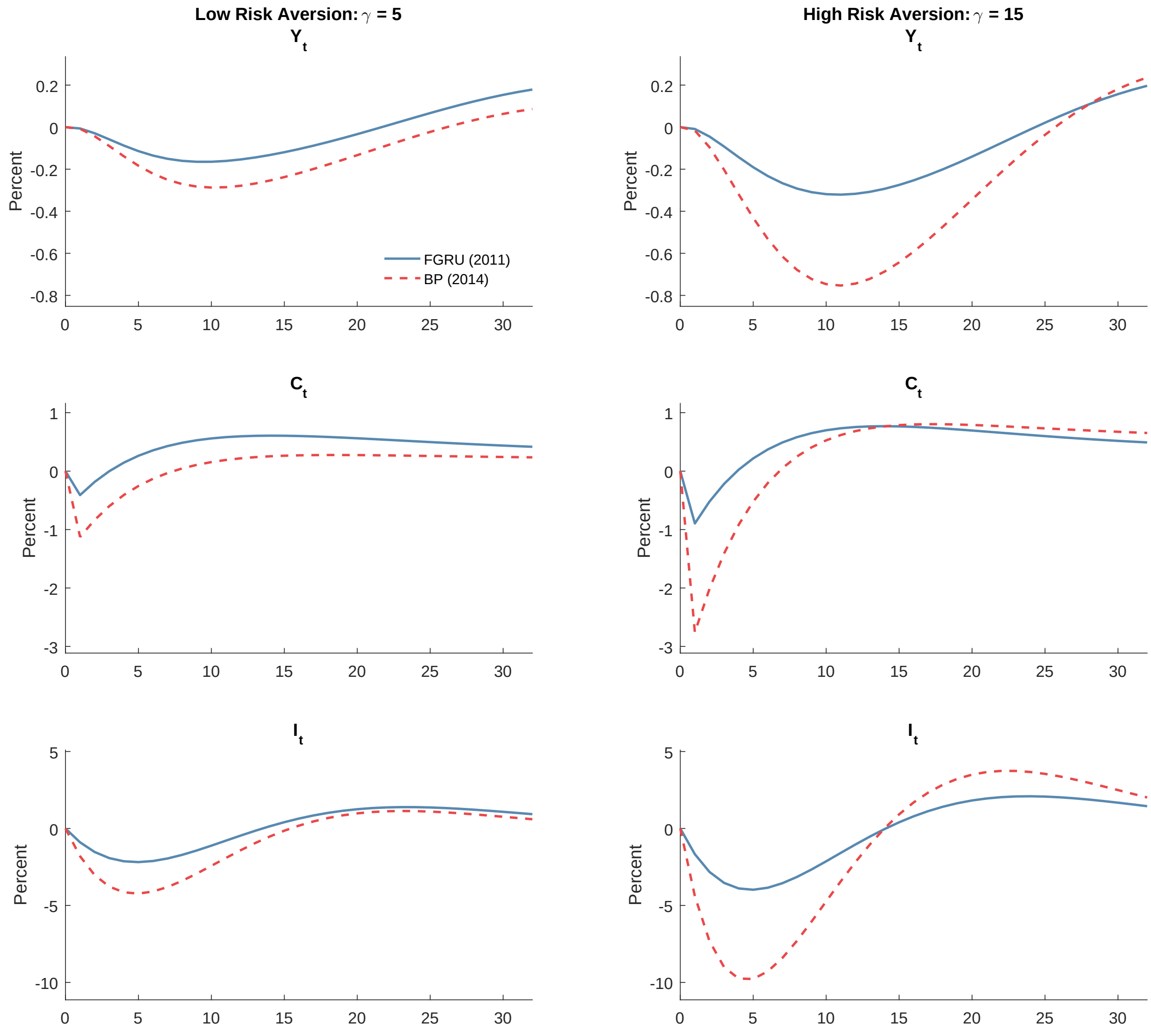


FIGURE 3: **Impulse Response Function to a Volatility Shock Interest Rates – FGRU (2011) vs BP (2014)**

Impulse responses are for a one standard deviation shock to the conditional volatility in interest rate when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).

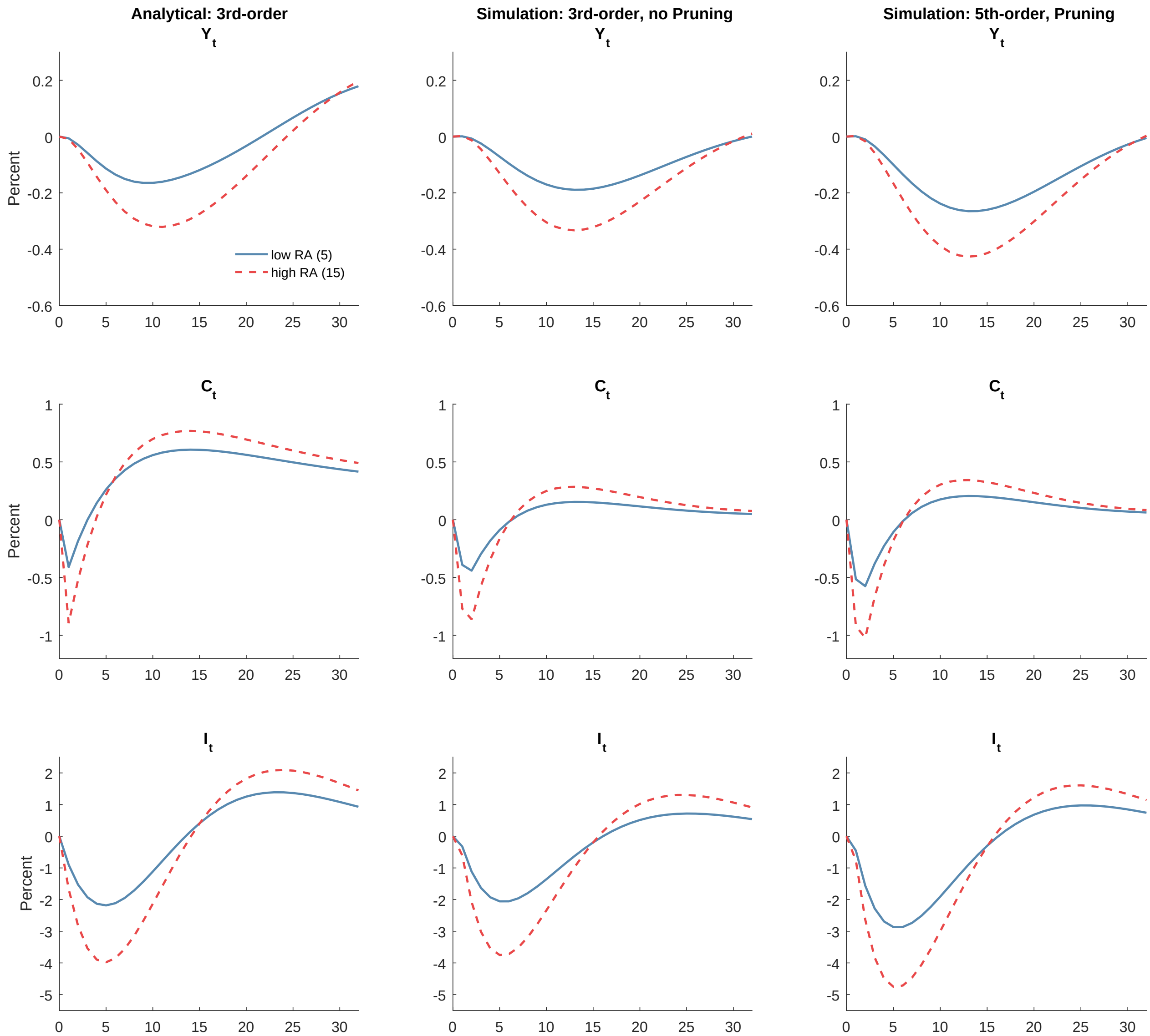


FIGURE 4: **(Unconditional) Impulse Responses to a Volatility Shock in Interest Rates – FGRU (2011)**

Impulse responses are for a one standard deviation shock when the model is approximated up to third order (first and second columns) and to the fifth order (last column). The IRFs in the first column must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). The IRFs in the second to third column must be interpreted as percentage deviations from the ergodic mean in the absence of shocks (EMAS).

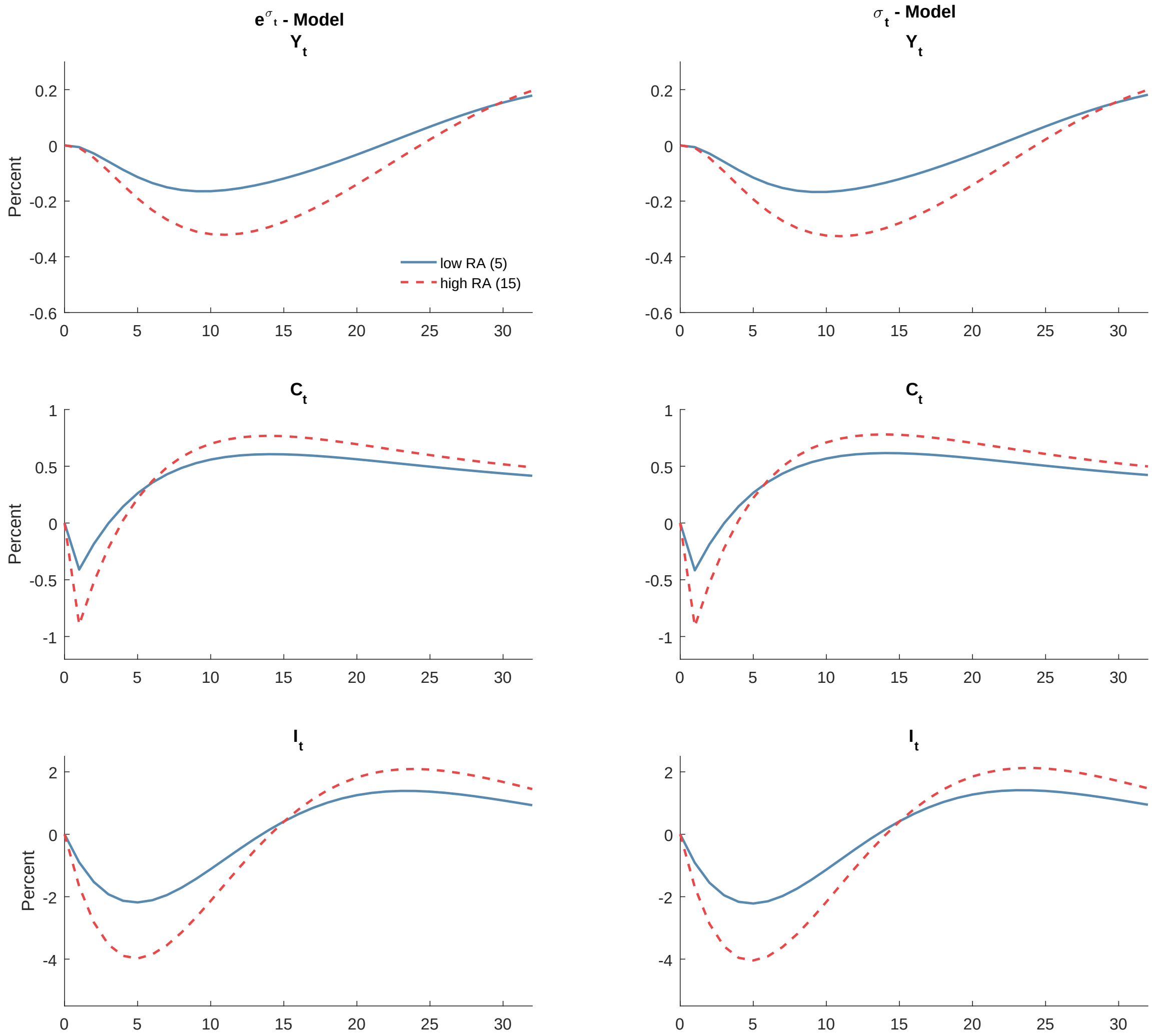
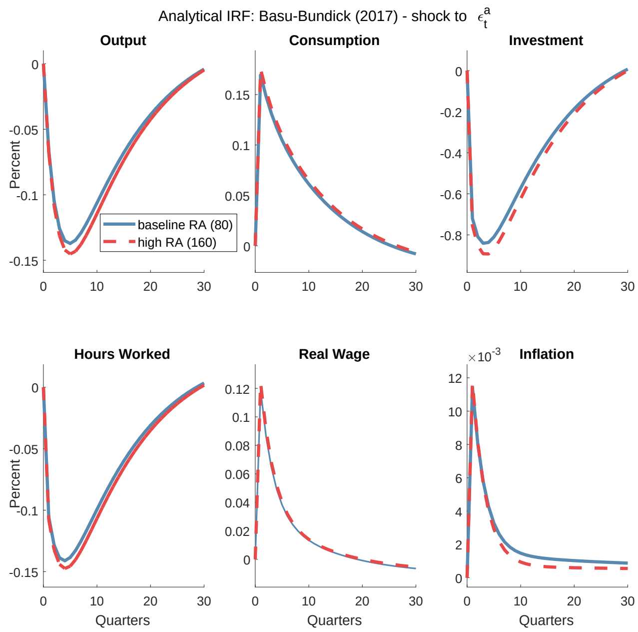
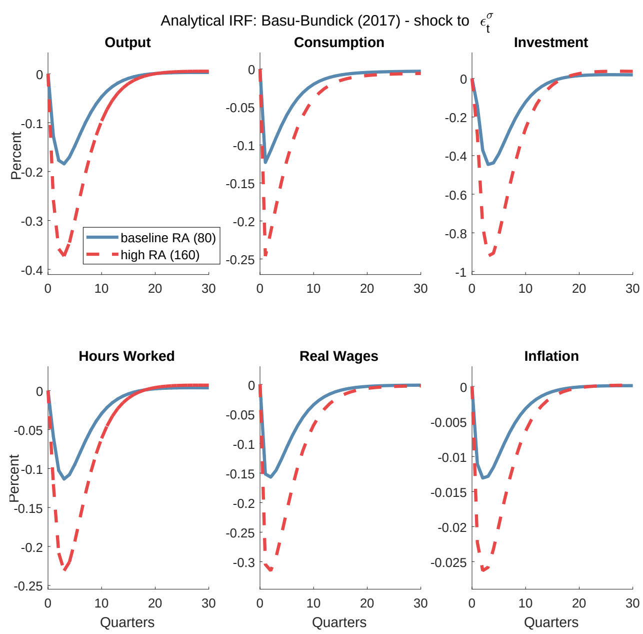


FIGURE 5: **(Unconditional) Impulse Response Function to a Volatility Shock Interest Rates – FGRU (2011)**

Impulse responses are for a one standard deviation shock to the conditional volatility in interest rate when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). The first column focuses on the specification that is commonly used in macroeconomics: the functional form is $m(x_t) \equiv \exp(x_t)$ with $x_t = \sigma_{r,t}$. The second column focuses the typical specification used in finance papers: the functional form is $m(x_t) \equiv \sqrt{(x_t)}$ and $x_t = \sigma^2_{r,t}$. In all cases the term x_t follows an exogenous stochastic AR(1) process.



(a) Response to level shock.



(b) Response to volatility shock.

FIGURE 6: **(Unconditional) Impulse Response Function – Basu and Bundick (2017)**: This figure plots the impulse responses for a one standard deviation shock to the (i) level, and (ii) volatility of the exogenous process for household discount factors. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).

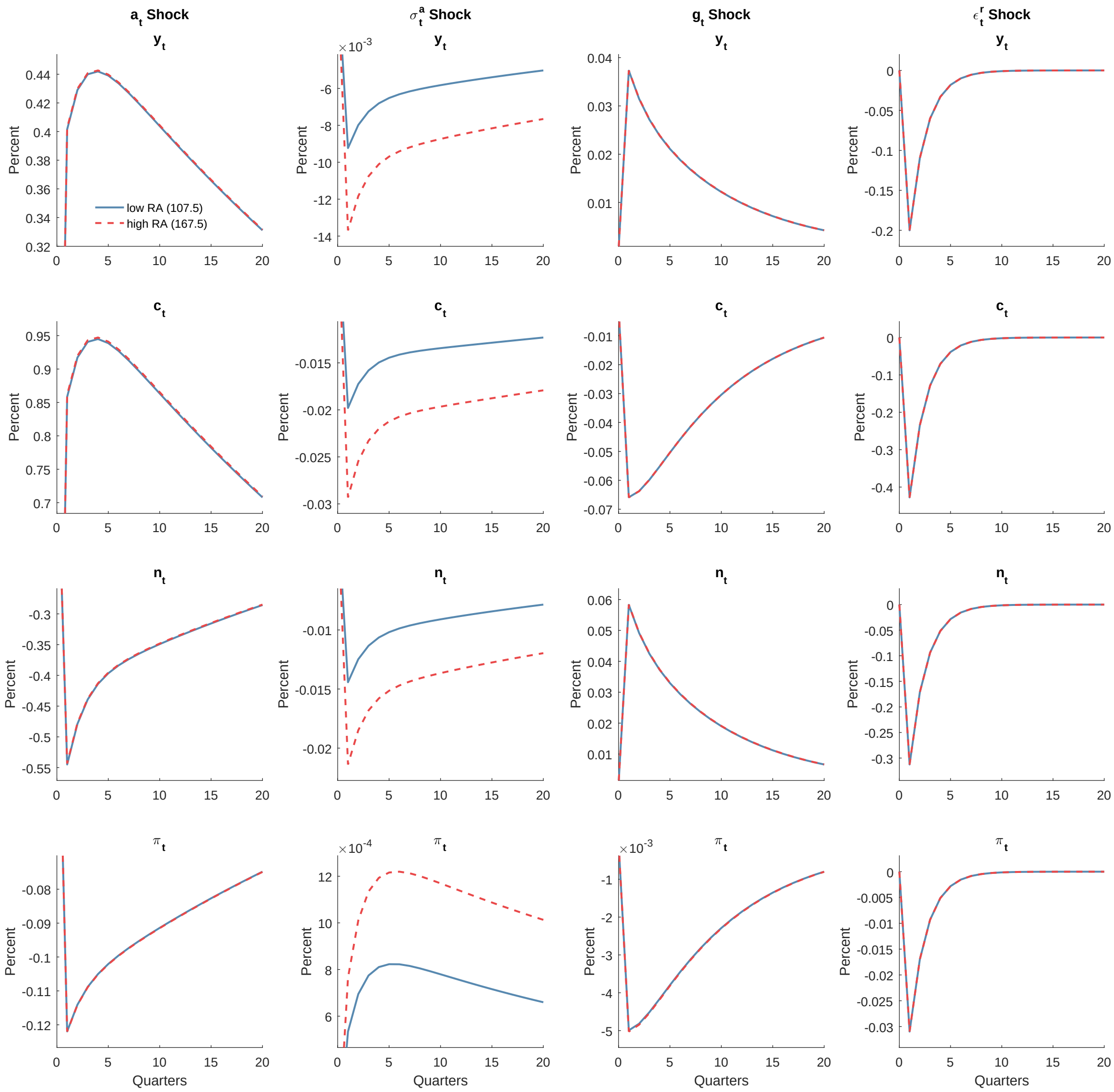


FIGURE 7: **(Unconditional) Impulse Response Function – Andreasen (2012)**

This figure plots the impulse responses for a one standard deviation shock to the (i) technology level (ii) conditional volatility in technology (iii) government consumption (iv) nominal interest rate. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).

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A Perturbation Methods and Generalized Impulse Response Function

This appendix includes a more detailed discussion of the solution of the model and the explanation of how we compute the IRFs and the variance decomposition of the model.

To judge the importance of volatility shocks for business cycle moments, and their interaction with risk aversion, our analysis relies on perturbation methods. Perturbation methods were first extensively applied to dynamic stochastic models by Judd (1998).

Our investigation faces a number of computational challenges. First, we are interested in the implications of a volatility increase while keeping the level of the variable constant. We thus have to consider a third-order Taylor expansion of the solution of the model, see e.g. Schmitt-Grohe and Uribe (2004), Fernandez-Villaverde et al. (2011) and Fernandez-Villaverde et al. (2015). Indeed, in a first-order approximation, stochastic volatility would not even play a role, since the policy rules of the representative agent follow a certainty equivalence principle. In the second-order approximation, only the product of the two innovations appears in the policy function. Only in the third-order approximation do the innovations to volatility play a role by themselves.[18]

Second, higher order perturbation solutions tend to explode due to the accumulation of terms of increasing order. For example, in a second order approximated solution, the quadratic term at time t will be raised to the power of two in the quadratic term at $t+1$, thus resulting in a quartic term, which will become a term of order 8 at $t+2$ and so on. As a solution, we adopt the pruning scheme described in Andreasen et al. (2016). This pruning scheme augments the state space to keep track of first to third order terms and uses the Kronecker product of the first and second order terms to compute the third order term. In contrast, Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) use a IRF-pruning scheme were all higher order terms are based on the first-order terms. Also, whereas in Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) the IRF-pruning scheme differs from the scheme used for simulations, we use the same pruning for both IRFs and simulations.

Third, computing IRFs in a nonlinear environment is somewhat involved, since the IRFs are not invariant to rescaling and to the previous history of shocks. To circumvent this problem, we consider the generalized impulse response function (GIRF) proposed by Koop et al. (1996). In particular, we compute GIRFs at the true ergodic mean using the methods proposed in Andreasen et al. (2016). In contrast, Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) start the IRFs at the ergodic mean in the absence of shocks (EMAS). However, the analytical expression for the ergodic mean is available only for the third-order (or lower) pruned state space described in Andreasen et al. (2016). Thus in the second and third columns in Figure 4 we compute the IRFs at the EMAS. In particular, we first simulate the model, starting from the ergodic mean (obtained analytically using a third-order pruned state space), for 2,096 periods. We disregard the first 2,000 periods as a burn-in and use the last 96 periods to compute the IRFs. In period 2,001 we set the realization of one of the shocks (productivity, the country spreads and its volatility) to one. We repeat this exercise 200 times to obtain the mean of the IRFs over the 200 simulations. As we mentioned in the main text as well as in the next section, since the data come in quarterly frequency, we build quarters of data from the model-simulated monthly IRFs.

[18] Recently, de Groot (2016) shows that to risk-correct the constant term for the standard deviation of stochastic volatility innovations (a.k.a. vol of vol) a fourth (or sixth, depending on the functional form of the volatility process) order expansion is further needed. de Groot (2016) shows that this risk-correction has important consequences for the bond and equity risk premia as well as for understanding the welfare cost of business cycle fluctuations.

Fourth, to judge the importance of risk shocks for business cycle moments, it is instructive to consider a variance decomposition. However, computing a variance decomposition is complicated because, with a third-order approximation to the policy function and its associated nonlinear terms, we cannot neatly divide total variance among the three shocks as we would do in the linear case. Thus, to gauge the relative importance of shocks we follow Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014), and we simulate the model with only a subset of the shocks. More precisely, we simulate the model, starting from the ergodic mean, for 96 periods. We hit the equilibrium system with a subset of the shocks. As we mentioned in the main text as well as in the next section, since the data come in quarterly frequency, we build quarters of data from the model-simulated variables, and we H-P filter them. The simulations are always restarted at this point after 96 periods and there is no burn-in. We repeat this exercise 200 times to obtain the mean of the moments over the 200 simulations. Table 4 check the stability of our simulations.

B The FGRU (2011) Model: Additional Details

B.1 Steady State, EMAS, and Ergodic Mean

This appendix compares the deterministic steady state, the ergodic mean in the absence of shocks (EMAS), and the ergodic mean for the Fernandez-Villaverde et al. (2011) model. It is in fact well know that time-varying volatility moves the ergodic distribution of the endogenous variables of the model away from their deterministic steady state. The theoretical mean are based on the third-order pruned state space of Andreasen et al. (2016). We use the term EMAS for Fernandez-Villaverde et al. (2011)'s concept of "[s]tarting from the ergodic mean and in the absence of shocks" (p. 10 in their technical appendix). The EMAS is the fixed point of the third order approximated policy functions in the absence of shocks. Sometimes, it is referred to as the "stochastic steady state" (e.g. Juillard and Kamenik, 2005), because it is the point of the state space where, in absence of shocks in that period, agents would choose to remain although they are taking future volatility into account.

Table B.1 compares steady state, ergodic mean, and EMAS in the original model of Fernandez-Villaverde et al. (2011). Results for the Born and Pfeifer (2014) re-calibrated model after correcting for time aggregation are available upon request.

Table B.1: Ergodic Mean FGRU (2011)

This table reports the steady-state values, the analytical ergodic means, and the simulated ergodic means in the absence of shocks for the FGRU (2011) model. We consider also the model with Epstein-Zin preferences when risk aversion equals the inverse of the elasticity of substitution.

		Analytical Ergodic Mean		Simulated EMAS	
	Steady State	FGRU	FGRU with EZ	FGRU	FGRU with EZ
D_t	4.00	2.09	2.09	2.55	2.55
K_t	3.29	3.31	3.31	3.29	3.29
C_t	0.88	0.91	0.91	0.89	0.89
H_t	0.00	0.00	0.00	0.00	0.00
Y_t	1.05	1.06	1.06	1.05	1.05
I_t	-0.98	-0.97	-0.97	-0.98	-0.98
NX_t^Y	0.03	0.01	0.01	0.02	0.02
CA_t	0.00	0.00	0.00	0.00	0.00

Table B.2: Parameters for FGRU (2011) model economy

This table reports the parameters used for the Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) models. These are the same values as in their original papers, and reported here for readers convenience. The high risk aversion scenario refers to a change in γ from 5 to 15, while leaving all other parameters unchanged.

	FGRU (2011)	BP (2014)
ψ	5.0000	5.0000
η	1000.0000	1000.0000
γ	5.0000	5.0000
β	0.9804	0.9804
δ	0.0140	1.0560
α	0.3200	0.3200
$\bar{D}$	18.8016	4.0000
φ	47.8376	95.0000
ϕ	0.0006	0.0010
r	0.02	0.02
ρ_x	0.9500	0.9500
σ_x	-3.2168	-4.1997
ρ_r	0.9700	0.9700
σ_r	-5.7100	-5.7100
ρ_{σ_r}	0.9400	0.9400
η_r	0.4600	0.4600
ρ_{tb}	0.9500	0.9500
$\rho_{\sigma_{tb}}$	-8.0600	-8.0600
σ_{tb}	0.9400	0.9400
η_{tb}	0.1300	0.1300

B.2 Time Aggregation: Moments and IRFs

Fernandez-Villaverde et al. (2011) set up their model in monthly terms, but report results at quarterly frequency as most data are available at quarterly frequency only. We follow their approach, and we aggregate monthly output, consumption, investment to quarterly frequency by summing up monthly percentage deviations. The only exceptions are Figure 3 and Table 3 where we follow Born and Pfeifer (2014) and we aggregate by averaging percentage deviations of monthly flow variables.

For the moment computations, the percentage deviations are from the deterministic steady state. Following Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014), the quarterly variables are HP-filtered before using them to compute the moments.

For the impulse response functions (IRFs) the percentage deviations are from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).

C Model: Andreasen (2012)

C.1 Stochastic Volatility and Perturbation Solution

To compute impulse response functions in Section 3.3 we have relied on the method suggested by Andreasen et al. (2016). This approach is based on a pruned state-space system for non-linear DSGE models. The advantage of the pruned state-space system is that it delivers closed-form solutions for the impulse response functions, and it avoids the use of simulation. In particular, Andreasen et al. (2016) provide analytical expressions for the generalized impulse response function (GIRF) when the model is approximated up to third order. In Figure C.1 we investigate the effects of pruning and the order of approximation on our results.

[Insert Figure C.1 about here.]

The second column in Figure C.1 shows that our results are not affected by pruning. When the system is not "pruned", no analytical expression is available and we have to rely on simulation. In this case we follow Andreasen (2012) and define the impulse responses for y_t

$$GIRF(y_{t+l};k) \equiv E_t\left[y_{t+l};x_t,\{\widetilde{\varepsilon}_{t+i}\}_{i=1}^{l}\right] - E_t\left[y_{t+l};x_t,\{\varepsilon_{t+i}\}_{i=1}^{l}\right]$$

where $\widetilde{\varepsilon}_{t+1} = \varepsilon_{t+1} + k$ and $\widetilde{\varepsilon}_{t+i} = \varepsilon_{t+i}$ for $i = 0, 1, 2, 3, \ldots, l$. The expectations are evaluated by simulation using 10,000 draws. Here, ε_{t+i} are drawn from its specified distribution while each value of x_t is taken from a sufficient long sample path of x_t. Impulse responses for a one-standard-deviation shock is given by $k = 1$ while $k = 3$ corresponds to a three-standard-deviation shock.

The third column in Figure C.1 shows that our results are unaffected by solving for decision rule up to fifth-order. To handle higher than third-order approximations (and to overcome the computational challenges) we rely on the approach developed by Fernandez-Villaverde and Levintal (2016).

C.2 Stochastic Volatility: Alternative Functional Forms

Figures C.2 and C.3 show how our results change depending on the functional form used for the stochastic volatility process. This analysis is important because the finance and macro literature have largely specified stochastic volatility processes differently. One can re-write Eq. (17)–(18) as follows:

$$a_{t+1} = (1-\rho_a)\, a_{ss} + \rho_a a_t + m(\sigma_{a,t+1})\varepsilon_{a,t+1}$$
$$x_{t+1} = (1-\rho_x)\, x + \rho_x x_t + \varepsilon_{x,t+1}$$

where now the innovations are scaled by $m\,(x_{t+1})$, but we leave unspecified the functional form of $m\,(\cdot)$. The previous analysis focused on the functional form $m\,(\cdot) \equiv \exp\,(\cdot)$ and $x_{t+1} = \sigma_{a,t+1}$. This specification is commonly used in macroeconomics. In contrast, finance papers like to use $m\,(\cdot) \equiv \sqrt{(\cdot)}$ and $x_{t+1} = \sigma_{a,t+1}^2$.[19] Figure C.2 shows that our results are not affected by either one of this functional choice.

[19] The benefit of the $m\,(\cdot) \equiv \sqrt{(\cdot)}$ specification is that the stochastic process is still conditionally normal and can be exploited to generate a conditionally log-normal linear approximation that accounts for risk as in Campbell and Shiller (1988). The drawback of this functional form is that it is possible to get a negative standard deviation. The functional form $m\,(\cdot) \equiv \exp\,(\cdot)$ ensures the standard deviation remains strictly positive but, as pointed out by Andreasen (2010), has the drawback that the level of the process does not have any moments.

[Insert Figures C.2 and C.3 about here.]

Another natural candidate to model time-varying volatility is the ARCH model proposed by Engle (1982) and its various generalizations. In a GARCH model, the conditional volatility is a function of lagged volatility and lagged squared residuals of the technology process. Thus, a GARCH process is not driven by separate innovations relative to the technology process. Figure C.3 shows the impulse responses to a technology shock with GARCH. Importantly, the figure shows that when the volatility has no separate innovations relative to the process for the level than risk aversion does not affect macro dynamics.

Figures

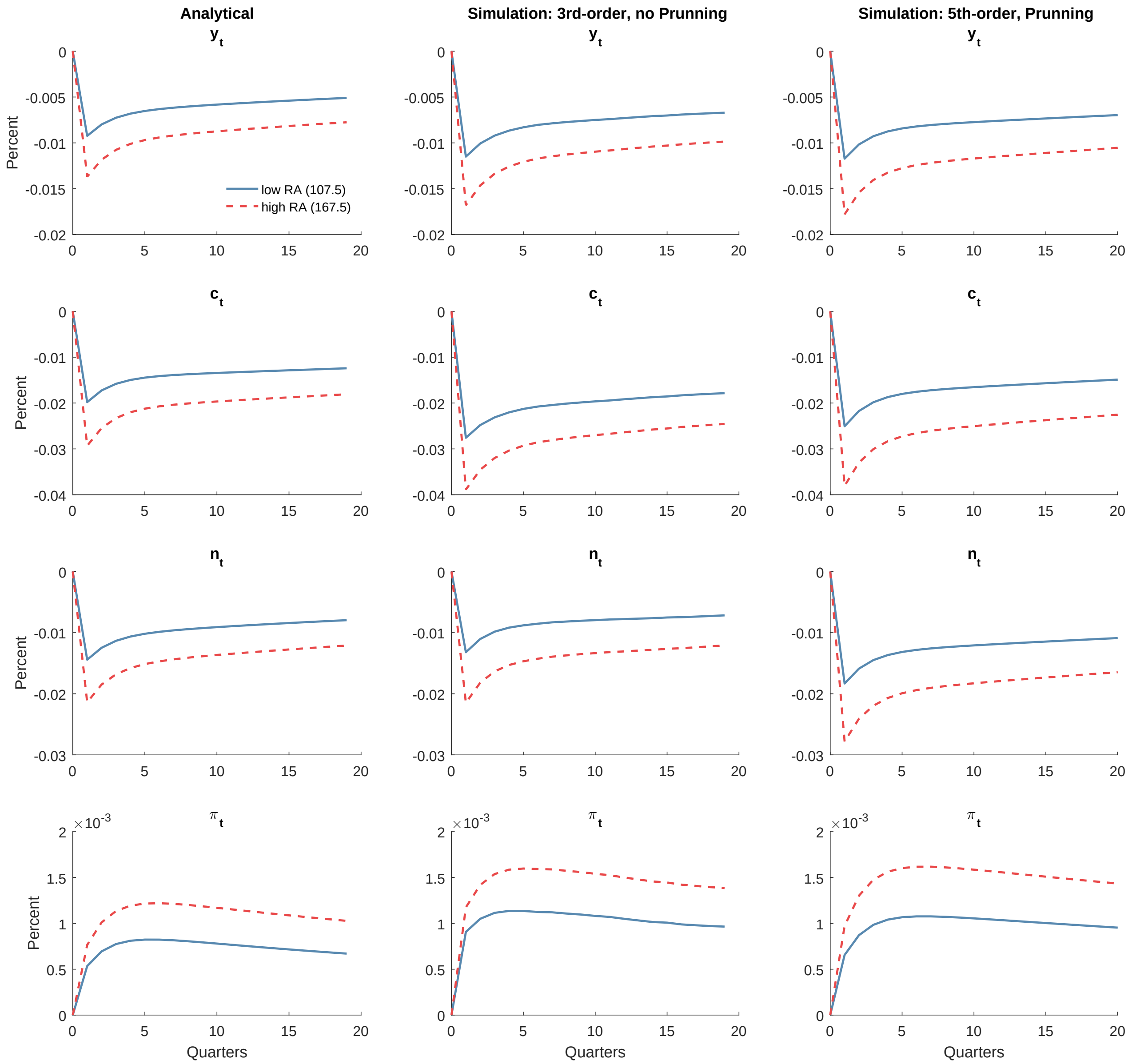

Figure C.1: **(Unconditional) Impulse Responses to a Volatility Shock in Technology – Andreasen (2012)**

Impulse responses are for a one standard deviation shock to the conditional volatility in technology when the model is approximated up to third order (first and second columns) and to the fifth order (last column).

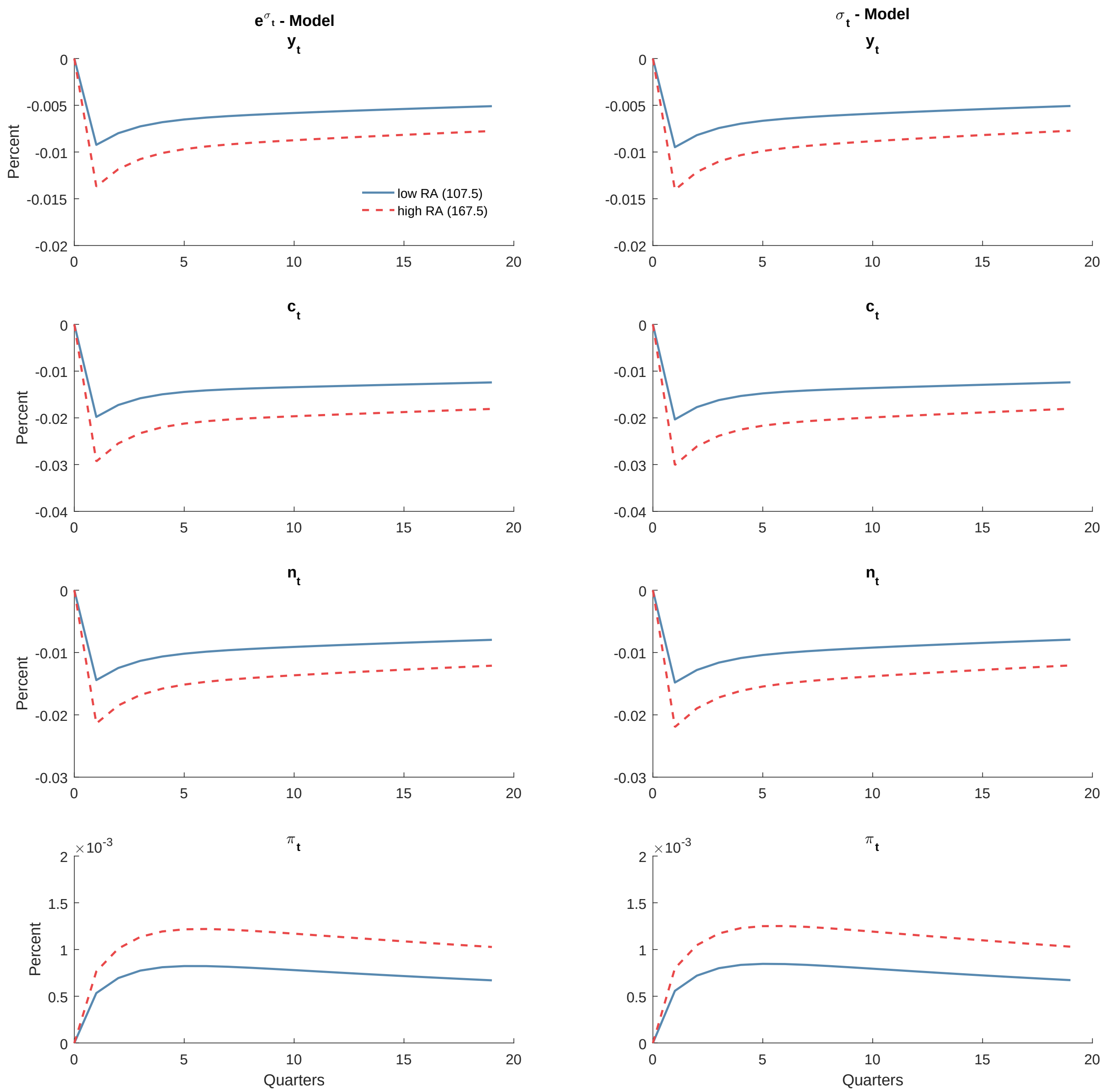

FIGURE C.2: **(Unconditional) Impulse Response Function to a Volatility Shock in Technology – Andreasen (2012)**

Impulse responses are for a one standard deviation shock to the conditional volatility in technology when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). The first column focuses on the specification that is commonly used in macroeconomics: the functional form is $m(x_t) \equiv \exp(x_t)$ with $x_t = \sigma_{a,t}$. The second column focuses the typical specification used in finance papers: the functional form is $m(x_t) \equiv \sqrt{(x_t)}$ and $x_t = \sigma^2_{a,t}$. In all cases the term x_t follows an exogenous stochastic AR(1) process.

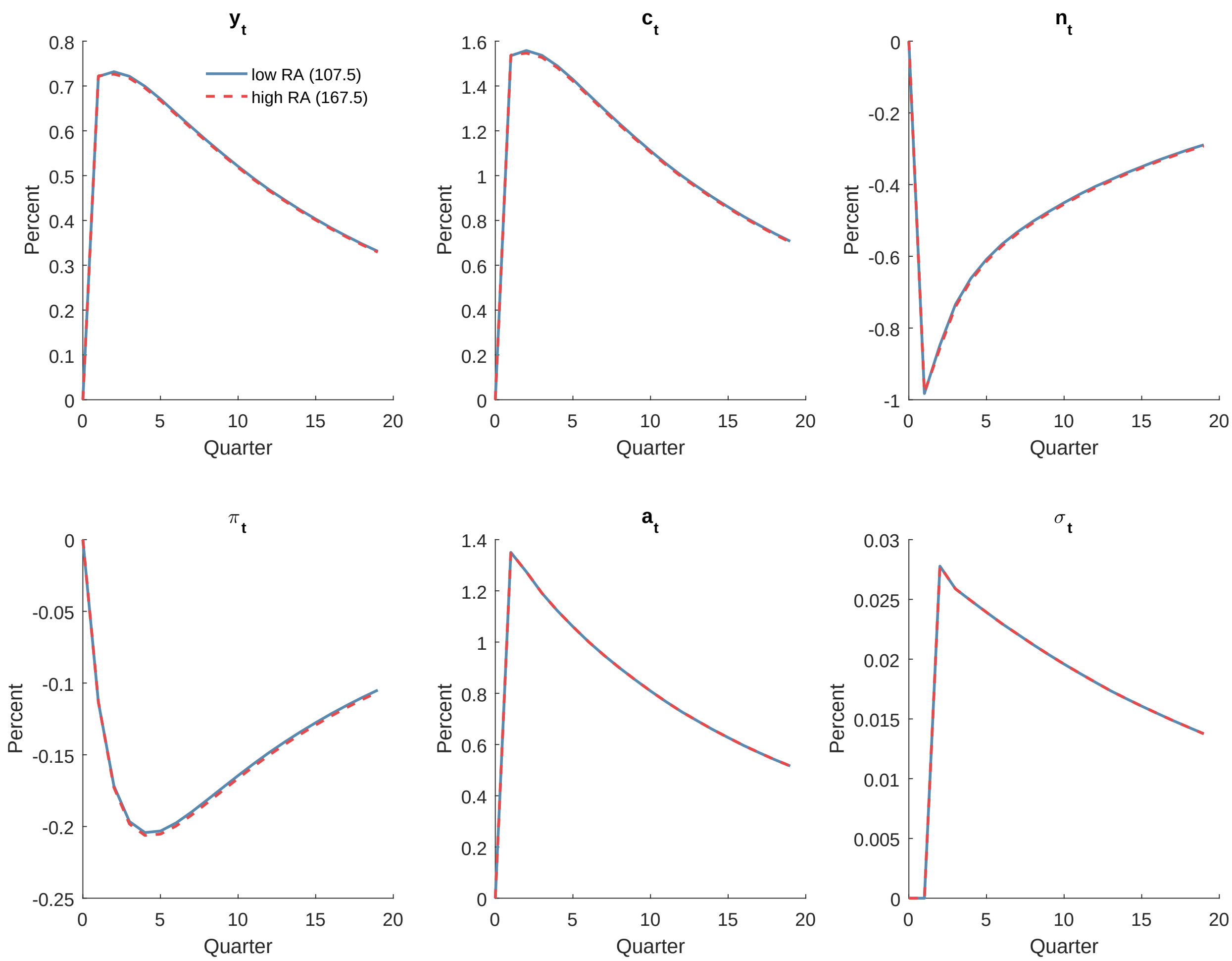

Figure C.3: **Impulse Responses to a Technology Shock with GARCH – Andreasen (2012)**

Impulse responses are for a one standard deviation shock to the conditional volatility in technology when the model is approximated up to third order.